TOWNSHIP OF UNION PUBLIC SCHOOLS

Honors Calculus
Adopted March 17, 2020

District Mission Statement

The Township of Union Board of Education believes that every child is entitled to an education, designed to meet his or her individual needs, in an environment that is conducive to learning. State standards, federal and state mandates, and local goals and objectives, along with community input, must be reviewed and evaluated on a regular basis to ensure that an atmosphere of learning is both encouraged and implemented. Furthermore, any disruption to or interference with a healthy and safe educational environment must be addressed, corrected, or, when necessary, removed, in order for the district to maintain the appropriate educational setting.

District Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is the formulation of a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

Throughout this course, students will investigate both differential and integral calculus. They will compute derivatives of polynomial, rational, trigonometric, exponential, and logarithmic functions and will apply the concept of a derivative as a rate of change to real world situations. Students will also compute antiderivatives of polynomial, rational, trigonometric, exponential, and logarithmic functions and apply the concept of an integral as a net change to real world situations. Students will analyze all functions analytically, numerically using a table, and graphically.

Recommended Textbooks:

Calculus of a Single Variable by Larson, Hostellar, Edwards (Houghton Mifflin)

Curriculum Units

Unit 1: Functions and Graphs
Unit 2: Limits
Unit 3: Finding Derivatives/Applications of Derivatives
Unit 4: Antiderivatives/Integrals
Unit 5: Applications of Antiderivatives

Pacing Guide

Unit 1: Functions and Graphs	5 weeks
Unit 2: Limits	6 weeks
Unit 3: Finding Derivatives/Applications of Derivatives	17 weeks
Unit 4: Antiderivatives/Integrals	7 weeks
Unit 5: Applications of Antiderivatives	7 weeks

Unit 1: FUNCTIONS & GRAPHS

NJSLS	NJSLS Content	Learning Activities
12.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
12.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.	
12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
12.A- SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
12.A-REI.B.3	Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.	
12.A-REI.B.4	Solve quadratic equations in one variable.	
12.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
12.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.	

Unit 1 Proficiencies
Students will be able to:

- Use knowledge of even and odd functions to predict graphical behavior
- Use knowledge of leading exponents to discuss end behavior of a function
- Use knowledge of intercepts and asymptotes to describe a graph over an interval
- Demonstrate removable and nonremovable discontinuites graphically and algebraically
- Explore properties of oscillating graphs
- Review trigometric functions and the unit circle

Suggested Differentiation for Unit 1

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

Curriculum Resources

- Textbook - Calculus of a Single Variable

- College Board AP Calculus AB (https://secure-media.collegeboard.org/apc/ap-calculus-course-description.pdf)

Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects Considering intercepts, asymptotes, end behavior and symmetry construct a reasonably accurate graph Use graphing calculator to verify conjectures Show that if a function is continuous and has no zeros it is always positive or always negative Compare relative magnitudes of functions and their rates of change using exponential, logarithmic and polynomial samples

Interdisciplinary Connections/Technology

Physics: Rates of change can be applied to motion, electricity, heat, light, and astronomy.
Biology: Calculus is used to compute birth and death rates or population growth/decline.

Unit 2: LIMITS

NJSLS	NJSLS Content	Learning Activities
12.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
12.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.	
12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
12.A- SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	
12.A-REI.B.3	Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.	
12.A-REI.B.4	Solve quadratic equations in one variable.	
12.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.	
12.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.	

Unit 2 Proficiencies

Students will be able to:

- Identify the limit value of a function as x approaches a given value (graphically, analytically, and numerically).
- Identify the equations of both horizontal and vertical asymptotes.
- Calculate limit values as *x* approaches infinity.
- Determine if a function is continuous both at a point and over an interval.
- Calculate an average rate of change over an interval (graphically, analytically, and numerically).
- Calculate an instantaneous rate of change at a point (analytically).

Suggested Differentiation for Unit 2

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

Curriculum Resources

- Textbook - Calculus of a Single Variable

- College Board AP Calculus AB (https://secure-media.collegeboard.org/apc/ap-calculus-course-description.pdf)

Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects

Interdisciplinary Connections/Technology

Physics: Rates of change can be applied to motion, electricity, heat, light, and astronomy.
Biology: Calculus is used to compute birth and death rates or population growth/decline.

Unit 3 : Derivatives/Application of Derivatives

NJSLS	NJSLS Content	Learning Activities
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
12.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	
12.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function	
12.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.	
12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	

12.F-LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.	

Unit 3 Proficiencies

Students will be able to:

- Calculate an instantaneous rate of change of polynomial, rational, exponential, logarithmic, and trigonometric functions at a point.
- Calculate an instantaneous rate of change of a curve defined implicitly.
- Determine the equations of lines tangent and normal to a curve at a given point.
- Predict the behavior of a function based on its first and second derivatives.
- Find maximum and minimum points of functions using the derivative and its properties.
- Determine concavity and find points of inflection.
- Solve optimization problems.
- Apply the Mean Value Theorem.
- Use differentials to estimate change
- Use linearization to approximate values.
- Determine related rates.

Suggested Differentiation for Unit 3

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.

- Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
- Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.

- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

Curriculum Resources

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- Copies of past AP Calculus AB exams

Formative Assessments	**Summative Assessments**
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects Explore slope field and solution curves for differential equations Determine increasing, decreasing, concave up and concave down areas of a curve Solve related rates and optimization problems Write the equation of a tangent and normal line to a curve Solve velocity and acceleration problems

Interdisciplinary Connections/Technology
Economics: Calculus can reduce production costs and optimize profits. **Epidemiology**: The rate at which a disease spreads and how far it spreads can be modeled and analyzed using calculus. **Medicine**: Medicine dosage rates can be modeled using differential equation.

Unit 4 : Anti-derivatives/Integrals

NJSLS	NJSLS Content	Learning Activities
12.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
12.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression	
12.A-CED.A.4	Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations	
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context	
12.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function	
12.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	

12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
12.F-BF.A.1	Write a function that describes a relationship between two quantities.	
12.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology.	

Unit 4 Proficiencies

- Determine antiderivatives of functions and understand the connection between the area under the graph of the function and the integral.

Suggested Differentiation for Unit 4

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**

- Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.

- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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Formative Assessments	**Summative Assessments**
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects Do work problems using a constant variable force Apply integration to motion on a line Solve separable differential equations Solve word problems using integration as an accumulation of change from a rate of change Explore length of a smooth curve and area of a surface of revolution using integration

Interdisciplinary Connections/Technology
Biology: Calculus is used to compute birth and death rates or population growth/decline.
Meteorology: Computer algorithms rely on differential equations to analyze data and predict weather.

Unit 5 : Application of Anti-derivatives

NJSLS	NJSLS Content	Learning Activities
12.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
12.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression	
12.A-CED.A.4	Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations	
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context	
12.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function	
12.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	

12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
12.F-BF.A.1	Write a function that describes a relationship between two quantities.	
12.G-GMD.B.4	Identify the shapes of two-dimensional cross sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.	

Unit 5 Proficiencies

- Apply integrals as a net change in real world situations.
- Determine antiderivatives of functions and understand the connection between the area under the graph of the function and the integral.
- Use integrals to calculate volumes.

Suggested Differentiation for Unit 5

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.

- Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.

- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects Solve word problems using integration as an accumulation of change from a rate of change Explore length of a smooth curve and area of a surface of revolution using integration

Interdisciplinary Connections/Technology

Meteorology: Computer algorithms rely on differential equations to analyze data and predict weather.

Medicine: Medicine dosage rates can be modeled using differential equations.

Additional Suggested Modifications for Units

Below is an additional list of modifications and accommodations opportunities. This includes, but is not limited to,:

1. English Language Learners.
    a. Read written instructions.
    b. Model and provide examples
    c. Extended time on assessments when needed.
    d. Establish a non-verbal cue to redirect student when not on task.
    e. Students may use a bilingual dictionary.

English Language Development Standard 3: Language of Mathematics: English language learners communicate information, ideas and concepts necessary for academic success in the content area of mathematics.

2. Special Education/504 Students.
    a. Extended time on assessments when needed.
    b. Preferred seating to be determined by student and teacher.
    c. Provide modified assessments when necessary.
    d. Student may complete assessments in alternate setting when requested.
    e. Establish a non-verbal cue to redirect student when not on task.
    f. Maintain strong teacher / parent communication.
    g. Conversion chart

New Jersey Student Learning Standards - Technology

8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations

B. Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.

C. Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning.

E: Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.

F: Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

***See Guide for Technology Integration.**

Career Readiness Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

NJSLS 9.2 - Career Awareness, Exploration, and Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.