Name ____________________________________

Algebra II Summer Packet

The Algebra II teaching staff at Union High School would like to welcome you. We are looking forward to working with you next school year. The following review was created to help you prepare for the Algebra II course you will be taking this fall. This packet includes concepts that are expected to be mastered before beginning the Algebra II curriculum. The topics covered in the packet are the basic skills necessary to be successful in Algebra II.

Algebra II teachers will be collecting the packet on the first Friday of school. There will be a graded assessment the following week.

Students may use any resources available to them to complete this packet. The websites Math.com and PHSchools.com are two helpful resources.

Be sure to spend the time needed to do a quality job on this packet. Show your work for each problem, so you and your teacher can locate any mistakes. In that way, you can learn from these mistakes. You many use a calculator where needed, but write down your calculations.

Enjoy your summer vacation and keep your education moving forward during this break.

1-5 Reteaching

Solving Inequalities

As with an equation, the solutions of an inequality are numbers that make it true. The procedure for solving a linear inequality is much like the one for solving linear equations. To isolate the variable on one side of the inequality, perform the same algebraic operation on each side of the inequality symbol.

The **Addition and Subtraction Properties of Inequality** state that adding or subtracting the same number from both sides of the inequality does not change the inequality.

If $a < b$, then $a + c < b + c$.

If $a < b$, then $a - c < b - c$.

The **Multiplication and Division Properties of Inequality** state that multiplying or dividing both sides of the inequality by the same *positive* number does not change the inequality.

If $a < b$ and $c > 0$, then $ac < bc$.

If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Problem

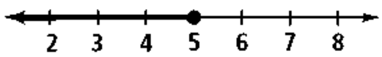
What is the solution of $3(x + 2) - 5 \leq 21 - x$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$3x + 6 - 5 \leq 21 - x$	Distributive Property
$3x + 1 \leq 21 - x$	Simplify.
$4x + 1 \leq 21$	Addition Property of Inequality
$4x \leq 20$	Subtraction Property of Inequality
$x \leq 5$	Division Property of Inequality

To graph the solution, locate the boundary point. Plot a point at $x = 5$. Because the inequality is *"less than or equal to,"* the boundary point is part of the solution set. Therefore, use a closed dot to graph the boundary point. Shade the number line to the left of the boundary point because the inequality is *"less than."*

Graph the solution on a number line.



Exercises

Solve each inequality. Graph the solution.

1. $2x + 4(x - 2) > 4$

2. $4 - (2x - 4) \geq 5 - (4x + 3)$

1-5 Reteaching (continued)

Solving Inequalities

The procedure for solving an inequality is similar to the procedure for solving an equation but with one important exception.

The Multiplication and Division Properties of Equality also state that, when you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol.

If $a < b$ and $c < 0$, then $ac > bc$.

If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

Problem

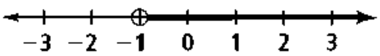
What is the solution of $2x - 3(x - 1) < x + 5$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$2x - 3(x - 1) < x + 5$	
$2x - 3x + 3 < x + 5$	Distributive Property
$-x + 3 < x + 5$	Simplify.
$-2x < 2$	Subtraction Property of Inequality
$x > -1$	Division Property of Inequality

The direction of the inequality changed in the last step because we divided both sides of the inequality by a negative number.

Graph the solution on a number line.



Exercises

Solve each inequality.

3. $x - 1 \leq -4(-2 - x)$

4. $7 - 7(x - 7) > -4 + 5x$

5. $7(x + 4) - 13 \geq 12 + 13(3 + x)$

6. $4x - 1 < 6x - 5$

1-6 Reteaching

Absolute Value Equations and Inequalities

Solving absolute value equations require solving two equations separately. Recall that for a real number *x*, $|x|$ is the distance from zero to *x* on the number line. The equation $|x| = p$ means that either $x = p$ or $x = -p$ because both are *p* units from 0.

Problem

What is the solution set for the equation $|5x+1| - 3 = 4$?

The first step in solving an absolute value equation is to isolate the absolute value on one side of the equal sign.

$\|5x+1\| - 3 = 4$	
$\|5x+1\| - 3 + 3 = 4 + 3$	Add 3 to each side.
$\|5x+1\| = 7$	Simplify.

Next, rewrite the absolute value as two equations and solve each of them separately.

$5x + 1 = 7$	or	$5x + 1 = -7$	Definition of absolute value
$5x = 6$	or	$5x = -8$	Addition Property of Equality
$x = \frac{6}{5}$	or	$x = -\frac{8}{5}$	Division Property of Equality

Notice that the same operations are performed in the same order on each of the two equations. However, do not try to "simplify" the process by solving a single equation. This leads to errors.

The solutions are $x = \frac{6}{5}$ or $x = -\frac{8}{5}$. Check each solution in the original equation:

Check

$\left\|5 \cdot \frac{6}{5} + 1\right\| - 3 = 4$	$\left\|5 \cdot \left(-\frac{8}{5}\right) + 1\right\| - 3 = 4$
$\|6+1\| - 3 = 4$	$\|-8+1\| - 3 = 4$
$4 = 4$ ✓	$4 = 4$ ✓

Exercises

Solve each absolute value equation. Check your work.

1. $|2x-3| - 4 = 3$

2. $|3x-6| + 1 = 13$

1-6 Reteaching (continued)

Absolute Value Equations and Inequalities

To solve an absolute value inequality, keep in mind that $|x|$ is the distance from zero to x on the number line. So, if $|x| < p$, then x is less than p units from 0, so

$$|x| < p \Rightarrow -p < x < p.$$

And, if $|x| > p$, then x is greater than p units from 0, so

$$|x| > p \Rightarrow x < -p \text{ or } x > p.$$

In this case, we need to rewrite the absolute value inequality as two separate inequalities. Do not try to combine them into one inequality.

Problem

What is the solution set for the inequality $|2x+3| > 11$?

Because the inequality is >, use $|x| > p \Rightarrow x < -p$ or $x > p$.

Begin by rewriting the absolute value as two equations and solve each of them separately.

$2x + 3 < -11$	or	$2x+3>11$	Rewrite as a compound inequality.
$2x < -14$	or	$2x > 8$	Subtract 3 from each side.
$x > -7$	or	$x > 4$	Divide each side by 2.

The solution set is $x < -7$ or $x > 4$.

Exercises

Complete the steps to solve the inequality $\left|\frac{x}{2} - 4\right| \leq 3$.

3. $\square \leq \frac{x}{2} - 4 \leq \square$ Rewrite as a compound inequality.

4. $\square \leq \frac{x}{2} \leq \square$ Add $\square$ to each part.

5. $\square \leq x \leq \square$ Multiply each part by $\square$.

6. What is the solution?

7. $5|y+3| < 15$

8. $|4b| - 3 > 9$

9. $\frac{1}{2}|2w-1| - 3 \geq 1$

10. $|3z-2| + 5 > 9$

2-3 Reteaching

Linear Functions and Slope-Intercept Form

You can use the slope-intercept form to write equations of lines.

- The slope-intercept formula is $y = mx + b$, where m represents the slope of the line, and b represents its y-intercept. The y-intercept is the point at which the line crosses the y-axis.
- The slope of a horizontal line is always zero, and the slope of a vertical line is always undefined.

Problem

What is the equation of the line that contains the point (3, −1) and has a slope of $-\frac{4}{3}$?

$-1 = \left(-\frac{4}{3}\right)(3) + b$	To find b, substitute the values $-\frac{4}{3}$ for m, 3 for x, and −1 for y into the slope-intercept formula.
$-1 = -4 + b$	Multiply.
$3 = b$	Add 4 to each side and simplify.
$y = -\frac{4}{3}x + 3$	Substitute $-\frac{4}{3}$ for m and 3 for b into the slope-intercept formula.

Exercises

Write an equation for each line.

1. $m = 4$; contains (3, 2)

2. $m = -2$; contains (4, 7)

3. $m = 0$; contains (3, 0)

4. $m = -1$; contains (−5, −2)

5. $m = 3$; contains (−2, −4)

6. $m = 0$; contains (0, −7)

7. $m = 8$; contains (5, 0)

8. $m = -1$; contains (0, 7)

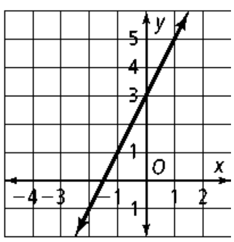
9. $m = 0$; contains (3, 8)

10. $m = 4$; contains (2, 5)

11. $m = 7$; contains (3, 2)

12. $m = -1$; contains (2, −6)

13.



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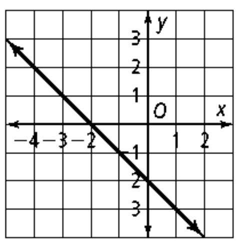

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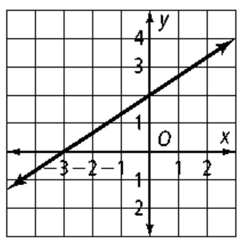


You can graph a linear equation if you know the slope and the y-intercept.

- Write the linear equation in slope-intercept form.
- Plot the y-intercept.
- Plot a second point using the slope.
- Draw a line through the two points.

Problem

What is the graph of $4x + 2y = 8$?

You can use the point-slope form to write equations of lines if you are given two points on the line. The point-slope form of a linear equation is:

$$y - y_1 = m(x - x_1)$$

Write the linear equation in slope-intercept form.

$4x + 2y = 8$ Original equation

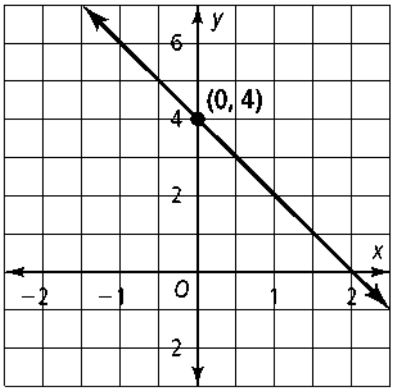
$2y = -4x + 8$

$y = -2x + 4$ Slope-intercept form

The slope is -2 and the y-intercept is $(0, 4)$.

Plot the y-intercept. Use the slope to plot a second point.

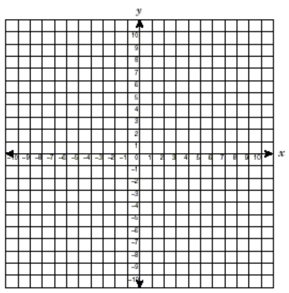
Then draw a line through the two points.



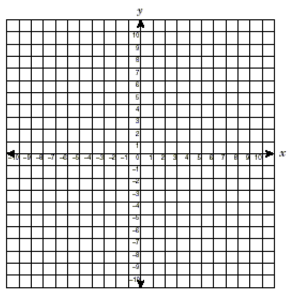
Exercises

Graph each equation.

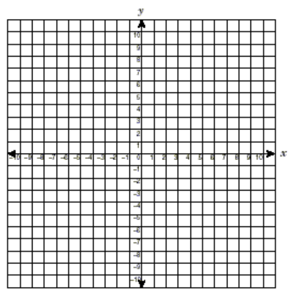
16. $-3x + 2y = 6$



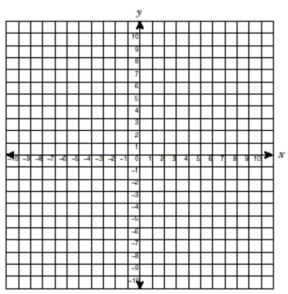
17. $3y + x = 3$



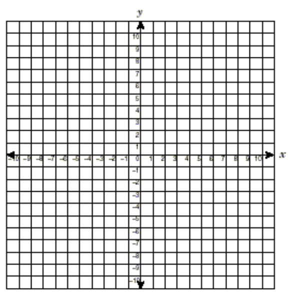
18. $3y - x = 2$



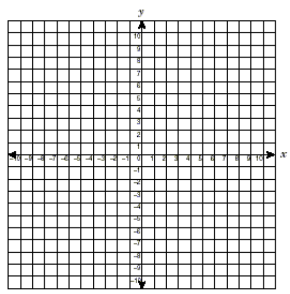
19. $-2x + 4y - 3 = 0$



20. $y + 7 = -2x$



21. $2y - 6 = 0$



2-4 **Reteaching**

What is the point-slope form of an equation of the line through (3, 4) and (5, –2)?

Find the slope. Substitute for each variable using the coordinates of the given

Problem

Let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (5, -2)$.	Identify each point.
So $x_1 = 3$, $y_1 = 4$, $x_2 = 5$, and $y_2 = -2$.	Identify x_1, x_2, y_1, and y_2.
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{5-3} = \frac{-6}{2} = -3$	Substitute the *x*- and *y*-values and simplify.

The slope is −3.

Write the equation of the line in point-slope form. Substitute one point and the slope into the point-slope formula. It does not matter which point is substituted into the point-slope formula.

Let $(x_1, y_1) = (3, 4)$.	Use either of the points as (x_1, y_1).
So $x_1 = 3$, $y_1 = 4$, and $m = -3$.	Identify the values of x_1, y_1, and *m*.
$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 4 = -3(x - 3)$	Substitute the values of x_1, y_1, and *m*.

An equation for the line in point-slope form is $y - 4 = -3(x - 3)$.

Exercises

Using point-slope form, write an equation of the line through each pair of points.

1. (6, −7) and (4, −1)

2. (3, 5) and (0, 7)

3. (−1, 3) and (2, 6)

4. (−1, −2) and (0, −1)

5. (−2, −5) and (8, −3)

6. (−1, 3) and (−7, −6)

7. (−3, 8) and (−2, 4)

8. (0, −2) and (9, 3)

2-4 Reteaching (continued)

More About Linear Equations

The slopes of parallel and perpendicular lines have special relationships. Parallel lines have the same slope. Lines that are perpendicular have slopes that are negative reciprocals of each other.

Problem

What is the equation for the line through the point (−1, 2) and parallel to $y = -2x + 4$? Write the equation in slope-intercept form.

Find the slope of the line. Parallel lines have the same slope, so the slope is −2.

Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y - 2 = -2(x + 1)$

Write the equation in slope-intercept form.

$y - 2 = -2(x + 1)$

$y - 2 = -2x - 2$

$\underline{+2} \qquad \underline{\quad +2}$

$y = -2x$

Problem

What is the equation for the line through the point (3, −1) and perpendicular to $y = 5x + 2$? Write the equation in slope-intercept form.

Find the slope of the line. Perpendicular lines have slopes that are negative reciprocals of each other. The negative reciprocal of 5 is $-\frac{1}{5}$.

Write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{1}{5}(x - 3)$

Write the equation in slope-intercept form.

$y + 1 = -\frac{1}{5}(x - 3)$

$y + 1 = -\frac{1}{5}x + \frac{3}{5}$

$\underline{-1} \qquad \underline{\quad -1}$

$y = -\frac{1}{5}x - \frac{2}{5}$

$-\frac{1}{5} \qquad \frac{3}{5}$

$\underline{-1} \qquad \underline{\quad -1}$

$y = -\frac{1}{5}x - \frac{2}{5}$

Exercises

Write an equation of each line in slope-intercept form.

9. through (−2, −2) and parallel to $y = -5x - 4$

10. through (−4, 1) and perpendicular to $y = -3x + 7$

11. through (0, 5) and parallel to $y = \frac{1}{2}x - 5$

12. through (0, −3) and perpendicular to $y = \frac{2}{3}x + 2$

2-8 RETEACHING – Two-Variable Inequalities

A **linear inequality** in two variables is an inequality whose graph is a region of the coordinate plane bounded by a line. This line is the **boundary**. If the boundary is included in the solution of the inequality, it is drawn as a solid line. If the boundary is not part of the solution of the inequality, it is drawn as a dashed line.

Problem

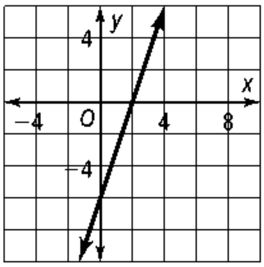
What is the graph of $6x - 2y \leq 12$?

$6x - 2y \leq 12$

$y \geq 3x - 6$

To graph the boundary line, write the inequality in slope-intercept form as if it were an equation.

The boundary line is solid if the inequality contains ≤ or ≥. The boundary line is dashed if the inequality contains, < or >. Graph the boundary line y = 3x – 6 as a solid line.



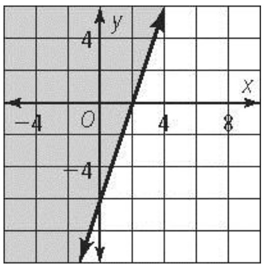
$0 \geq 3(0) - 6$

Since the boundary line does not contain the origin, substitute the point (0, 0) into the inequality.

$0 \geq -6$

Simplify. The resulting inequality is true.

Shade the region that contains the origin. If the resulting inequality were false, then you would shade the region that does not contain the origin.



Exercises

Graph each inequality.

1. $y > 2x$

2. $x + y < 4$

3. $y < x + 1$

4. $3x - 2 \leq 5x + y$

5. $x < -4$

6. $y \geq 5$

5-1 RETEACHING – Polynomial Functions

Problem

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^6$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

Step 3 Count the number of terms in the simplified polynomial. It has 5 terms.

Step 4 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (n) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
a (+3) is positive and n (6) is even, so the end behavior is up and up.

Exercises

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$ **2.** $15x^7 - 7$ **3.** $2x - 6x^2 - 9$