Intro to Precalculus with Statistics
Summer Assignment

Instructions: Review the entire Intro to Precalculus with Statistics Summer Packet. Complete numbers 1-125 on a separate sheet of paper. Be sure to show all work. This summer assignment is due to your teacher on the first day of school.

Chapter R Test Prep

Key Terms

R.1 set, elements (members), infinite set, finite set, Venn diagram, disjoint sets

R.2 number line, coordinate system, coordinate, power or exponential expression (exponential), exponent, base, absolute value

R.3 algebraic expression, term, coefficient, like terms, polynomial, polynomial in x, degree of a term, degree of a polynomial, trinomial, binomial, monomial, descending order, FOIL method

R.4 factoring, factored form, prime polynomial, factored completely, factoring by grouping

R.5 rational expression, domain of a rational expression, lowest terms, complex fraction

R.7 radicand, index of a radical, principal nth root, like radicals, unlike radicals, rationalizing the denominator, conjugates

New Symbols

$\{\ \}$	set braces
$\in$	is an element of
$\notin$	is not an element of
$\{x \mid x$ **has property** $p\}$	set-builder notation
U	universal set
$\emptyset$, or $\{\ \}$	null (empty) set
$\subseteq$	is a subset of
$\nsubseteq$	is not a subset of
A'	complement of a set A
$\cap$	set intersection
$\cup$	set union
a^n	n factors of a
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\lvert a \rvert$	absolute value of a
$\sqrt{\ }$	radical symbol

Quick Review

Concepts	Examples
R.1 Sets	
Set Operations	
For all sets A and B, with universal set U:	Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3, 4\}$, and $B = \{3, 4, 6\}$.
The **complement** of set A is the set A' of all elements in U that do not belong to set A. $$A' = \{x \mid x \in U, x \notin A\}$$	$A' = \{5, 6\}$
The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A and set B. $$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$	$A \cap B = \{3, 4\}$
The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A or to set B. $$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$	and $A \cup B = \{1, 2, 3, 4, 6\}$

Concepts	Examples
R.2 Real Numbers and Their Properties	
Sets of Numbers	
Natural numbers	
$\{1, 2, 3, 4, \ldots\}$	$5, 17, 142$
Whole numbers	
$\{0, 1, 2, 3, 4, \ldots\}$	$0, 27, 96$
Integers	
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	$-24, 0, 19$
Rational numbers	
$\{\frac{p}{q} \mid p$ and q are integers and $q \neq 0\}$	$-\frac{3}{4}, -0.28, 0, 7, \frac{9}{16}, 0.66\overline{6}$
Irrational numbers	
$\{x \mid x$ is real but not rational$\}$	$-\sqrt{15}, 0.101101110\ldots, \sqrt{2}, \pi$
Real numbers	
$\{x \mid x$ corresponds to a point on a number line$\}$	$-46, 0.7, \pi, \sqrt{19}, \frac{8}{5}$
Properties of Real Numbers	
For all real numbers a, b, and c:	
Closure Properties	
$a + b$ **is a real number.** ab **is a real number.**	$1 + \sqrt{2}$ is a real number. $3\sqrt{7}$ is a real number.
Commutative Properties	
$a + b = b + a$ $ab = ba$	$5 + 18 = 18 + 5$ $-4 \cdot 8 = 8 \cdot (-4)$
Associative Properties	
$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$	$[6 + (-3)] + 5 = 6 + (-3 + 5)$ $(7 \cdot 6)20 = 7(6 \cdot 20)$
Identity Properties	
There exists a unique real number 0 such that	
$a + 0 = a$ and $0 + a = a.$	$145 + 0 = 145$ and $0 + 145 = 145$
There exists a unique real number 1 such that	
$a \cdot 1 = a$ and $1 \cdot a = a.$	$-60 \cdot 1 = -60$ and $1 \cdot (-60) = -60$
Inverse Properties	
There exists a unique real number $-a$ such that	
$a + (-a) = 0$ and $-a + a = 0.$	$17 + (-17) = 0$ and $-17 + 17 = 0$
If $a \neq 0$, there exists a unique real number $\frac{1}{a}$ such that	
$a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1.$	$22 \cdot \frac{1}{22} = 1$ and $\frac{1}{22} \cdot 22 = 1$
Distributive Properties	
$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	$3(5 + 8) = 3 \cdot 5 + 3 \cdot 8$ $6(4 - 2) = 6 \cdot 4 - 6 \cdot 2$

(continued)

Concepts	Examples
Order	
$a > b$ if a is to the right of b on a number line.	$7 > -5$
$a < b$ if a is to the left of b on a number line.	$0 < 15$
Absolute Value	
$\|a\| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$	$\|3\| = 3$ and $\|-3\| = 3$

R.3 Polynomials

Special Products

Concepts	Examples
Product of the Sum and Difference of Two Terms	
$(x + y)(x - y) = x^2 - y^2$	$(7 - x)(7 + x) = 7^2 - x^2 = 49 - x^2$
Square of a Binomial	
$(x + y)^2 = x^2 + 2xy + y^2$	$(3a + b)^2 = (3a)^2 + 2(3a)(b) + b^2 = 9a^2 + 6ab + b^2$
$(x - y)^2 = x^2 - 2xy + y^2$	$(2m - 5)^2 = (2m)^2 - 2(2m)(5) + 5^2 = 4m^2 - 20m + 25$

R.4 Factoring Polynomials

Factoring Patterns

Concepts	Examples
Difference of Squares	
$x^2 - y^2 = (x + y)(x - y)$	$4t^2 - 9 = (2t + 3)(2t - 3)$
Perfect Square Trinomial	
$x^2 + 2xy + y^2 = (x + y)^2$	$p^2 + 4pq + 4q^2 = (p + 2q)^2$
$x^2 - 2xy + y^2 = (x - y)^2$	$9m^2 - 12mn + 4n^2 = (3m - 2n)^2$
Difference of Cubes	
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$	$r^3 - 8 = (r - 2)(r^2 + 2r + 4)$
Sum of Cubes	
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$	$27x^3 + 64 = (3x + 4)(9x^2 - 12x + 16)$

R.5 Rational Expressions

Concepts	Examples
Operations	
Let $\frac{a}{b}$ and $\frac{c}{d}$ $(b \neq 0, d \neq 0)$ represent fractions.	
$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$	$\frac{2}{x} + \frac{5}{y} = \frac{2y + 5x}{xy}$ $\frac{x}{6} - \frac{2y}{5} = \frac{5x - 12y}{30}$
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ $(c \neq 0)$	$\frac{3}{q} \cdot \frac{3}{2p} = \frac{9}{2pq}$ $\frac{z}{4} \div \frac{z}{2t} = \frac{z}{4} \cdot \frac{2t}{z} = \frac{2zt}{4z} = \frac{t}{2}$

Concepts	Examples
R.6 Rational Exponents	
Rules for Exponents Let r and s be rational numbers. The following results are valid for all positive numbers a and b.	
$a^r \cdot a^s = a^{r+s}$ $\quad (ab)^r = a^r b^r$ $\quad (a^r)^s = a^{rs}$	$6^2 \cdot 6^3 = 6^5$ $\quad (3x)^4 = 3^4 x^4$ $\quad (m^2)^3 = m^6$
$\frac{a^r}{a^s} = a^{r-s}$ $\quad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$ $\quad a^{-r} = \frac{1}{a^r}$	$\frac{p^5}{p^2} = p^3$ $\quad \left(\frac{x}{3}\right)^2 = \frac{x^2}{3^2}$ $\quad 4^{-3} = \frac{1}{4^3}$
R.7 Radical Expressions	
Radical Notation Suppose that a is a real number, n is a positive integer, and $a^{1/n}$ is defined. $\sqrt[n]{a} = a^{1/n}$	$\sqrt[4]{16} = 16^{1/4} = 2$
Suppose that m is an integer, n is a positive integer, and a is a real number for which $\sqrt[n]{a}$ is defined. $a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$	$8^{2/3} = \left(\sqrt[3]{8}\right)^2 = \sqrt[3]{8^2} = 4$
Operations Operations with radical expressions are performed like operations with polynomials.	$\sqrt{8x} + \sqrt{32x} = 2\sqrt{2x} + 4\sqrt{2x} = 6\sqrt{2x}$ $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = 5 - 3 = 2$ $(\sqrt{2} + \sqrt{7})(\sqrt{3} - \sqrt{6}) = \sqrt{6} - 2\sqrt{3} + \sqrt{21} - \sqrt{42}$ FOIL; $\sqrt{12} = 2\sqrt{3}$
Rationalize the denominator by multiplying numerator and denominator by a form of 1.	$\frac{\sqrt{7y}}{\sqrt{5}} = \frac{\sqrt{7y}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35y}}{5}$

Chapter R Review Exercises

1. Use set notation to list all the elements of the set $\{6, 8, 10, \ldots, 20\}$.
2. Is the set $\{x \mid x$ is a decimal between 0 and $1\}$ finite or infinite?
3. *Concept Check True* or *false:* The set of negative integers and the set of whole numbers are disjoint sets.
4. *Concept Check True* or *false:* 9 is an element of $\{999\}$.

Determine whether each statement is true *or* false.

5. $1 \in \{6, 2, 5, 1\}$
6. $7 \notin \{1, 3, 5, 7\}$
7. $\{8, 11, 4\} = \{8, 11, 4, 0\}$
8. $\{0\} = \emptyset$

Let $A = \{1, 3, 4, 5, 7, 8\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7\}$, $D = \{1, 2, 3\}$, $E = \{3, 7\}$, *and* $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Determine whether each statement is true *or* false.

9. $\emptyset \subseteq A$ **10.** $E \subseteq C$ **11.** $D \not\subseteq B$ **12.** $E \not\subseteq A$

Refer to the sets given for Exercises 9–12. Specify each set.

13. A' **14.** $B \cap A$ **15.** $B \cap E$

16. $C \cup E$ **17.** $D \cap \emptyset$ **18.** $B \cup \emptyset$

19. $(C \cap D) \cup B$ **20.** $(D' \cap U) \cup E$ **21.** $\emptyset'$

22. *Concept Check True* or *false:* For all sets A and B, $(A \cap B) \subseteq (A \cup B)$.

For Exercises 23 and 24, let set $K = \left\{-12, -6, -0.9, -\sqrt{7}, -\sqrt{4}, 0, \frac{1}{8}, \frac{\pi}{4}, 6, \sqrt{11}\right\}$. *List all elements of K that belong to each set.*

23. Integers **24.** Rational numbers

For Exercises 25–28, choose all words from the following list that apply.

natural number whole number integer
rational number irrational number real number

25. $\frac{4\pi}{5}$ **26.** $\frac{\pi}{0}$ **27.** 0 **28.** $-\sqrt{36}$

Write each algebraic identity (true statement) as a complete English sentence without using the names of the variables. For instance, $z(x + y) = zx + zy$ *can be stated as "The multiple of a sum is the sum of the multiples."*

29. $\frac{1}{xy} = \frac{1}{x} \cdot \frac{1}{y}$ **30.** $a(b - c) = ab - ac$

31. $(ab)^n = a^n b^n$ **32.** $a^2 - b^2 = (a + b)(a - b)$

33. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ **34.** $|st| = |s| \cdot |t|$

Identify by name each property illustrated.

35. $8(5 + 9) = (5 + 9)8$ **36.** $4 \cdot 6 + 4 \cdot 12 = 4(6 + 12)$

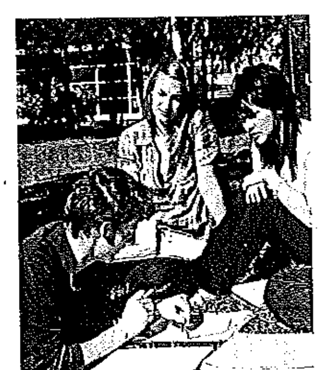
37. $3 \cdot (4 \cdot 2) = (3 \cdot 4) \cdot 2$ **38.** $-8 + 8 = 0$

39. $(9 + p) + 0 = 9 + p$ **40.** $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

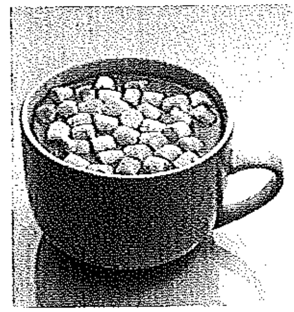
41. *Ages of College Undergraduates* The following table shows the age distribution of college students in 2008. In a random sample of 5000 such students, how many would you expect to be over 19?

Age	Percent
14–19	29
20–24	43
25–34	16
35 and older	12

Source: U.S. Census Bureau.



42. *Counting Marshmallows* In early 2011, there were media reports about students providing a correction to the following question posed on boxes of Swiss Miss Chocolate: *On average, how many mini-marshmallows are in one serving?*



$$3 + 2 \times 4 \div 2 - 3 \times 7 - 4 + 47 = \_\_\_\_\_$$

The company provided 92 as the answer. What is the *correct* calculation provided by the students? (*Source:* Swiss Miss Chocolate box.)

Simplify each expression.

43. $(-4 - 1)(-3 - 5) - 2^3$

44. $(6 - 9)(-2 - 7) \div (-4)$

45. $\left(-\frac{5}{9} - \frac{2}{3}\right) - \frac{5}{6}$

46. $\left(-\frac{2^3}{5} - \frac{3}{4}\right) - \left(-\frac{1}{2}\right)$

47. $\dfrac{6(-4) - 3^2(-2)^3}{-5[-2 - (-6)]}$

48. $\dfrac{(-7)(-3) - (-2^3)(-5)}{(-2^2 - 2)(-1 - 6)}$

Evaluate each expression for $a = -1$, $b = -2$, and $c = 4$.

49. $-c(2a - 5b)$

50. $(a - 2) \div 5 \cdot b + c$

51. $\dfrac{9a + 2b}{a + b + c}$

52. $\dfrac{3|b| - 4|c|}{|ac|}$

Perform the indicated operations.

53. $(3q^3 - 9q^2 + 6) + (4q^3 - 8q + 3)$

54. $2(3y^6 - 9y^2 + 2y) - (5y^6 - 4y)$

55. $(8y - 7)(2y + 7)$

56. $(2r + 11s)(4r - 9s)$

57. $(3k - 5m)^2$

58. $(4a - 3b)^2$

Perform each division.

59. $\dfrac{30m^3 - 9m^2 + 22m + 5}{5m + 1}$

60. $\dfrac{72r^2 + 59r + 12}{8r + 3}$

61. $\dfrac{3b^3 - 8b^2 + 12b - 30}{b^2 + 4}$

62. $\dfrac{5m^3 - 7m^2 + 14}{m^2 - 2}$

Factor as completely as possible.

63. $3(z - 4)^2 + 9(z - 4)^3$

64. $7z^2 - 9z^3 + z$

65. $z^2 - 6zk - 16k^2$

66. $r^2 + rp - 42p^2$

67. $48a^8 - 12a^7b - 90a^6b^2$

68. $6m^2 - 13m - 5$

69. $49m^8 - 9n^2$

70. $169y^4 - 1$

71. $6(3r - 1)^2 + (3r - 1) - 35$

72. $8y^3 - 1000z^6$

73. $xy + 2x - y - 2$

74. $15mp + 9mq - 10np - 6nq$

Factor each expression. (These expressions arise in calculus from a technique called the product rule that is used to determine the shape of a curve.)

75. $(3x - 4)^2 + (x - 5)(2)(3x - 4)(3)$

76. $(5 - 2x)(3)(7x - 8)^2(7) + (7x - 8)^3(-2)$

Perform the indicated operations.

77. $\frac{k^2+k}{8k^3} \cdot \frac{4}{k^2-1}$

78. $\frac{3r^3-9r^2}{r^2-9} \div \frac{8r^3}{r+3}$

79. $\frac{x^2+x-2}{x^2+5x+6} \div \frac{x^2+3x-4}{x^2+4x+3}$

80. $\frac{27m^3-n^3}{3m-n} \div \frac{9m^2+3mn+n^2}{9m^2-n^2}$

81. $\frac{p^2-36q^2}{p^2-12pq+36q^2} \cdot \frac{p^2-5pq-6q^2}{p^2+2pq+q^2}$

82. $\frac{1}{4y}+\frac{8}{5y}$

83. $\frac{m}{4-m}+\frac{3m}{m-4}$

84. $\frac{3}{x^2-4x+3}-\frac{2}{x^2-1}$

85. $\frac{p^{-1}+q^{-1}}{1-(pq)^{-1}}$

86. $\frac{3+\frac{2m}{m^2-4}}{\frac{5}{m-2}}$

Simplify each expression. Write the answer with only positive exponents. Assume all variables represent positive real numbers.

87. $\left(-\frac{5}{4}\right)^{-2}$

88. $3^{-1}-4^{-1}$

89. $(5z^3)(-2z^5)$

90. $(8p^2q^3)(-2p^5q^{-4})$

91. $(-6p^5w^4m^{12})^0$

92. $(-6x^2y^{-3}z^2)^{-2}$

93. $\frac{-8y^7p^{-2}}{y^{-4}p^{-3}}$

94. $\frac{a^{-6}(a^{-8})}{a^{-2}(a^{11})}$

95. $\frac{(p+q)^4(p+q)^{-3}}{(p+q)^6}$

96. $\frac{[p^2(m+n)^3]^{-2}}{p^{-2}(m+n)^{-5}}$

97. $(7r^{1/2})(2r^{3/4})(-r^{1/6})$

98. $(a^{3/4}b^{2/3})(a^{5/8}b^{-5/6})$

99. $\frac{y^{5/3}\cdot y^{-2}}{y^{-5/6}}$

100. $\left(\frac{25m^3n^5}{m^{-2}n^6}\right)^{-1/2}$

101. $\frac{(p^{15}q^{12})^{-4/3}}{(p^{24}q^{16})^{-3/4}}$

102. Simplify the product $-m^{3/4}(8m^{1/2}+4m^{-3/2})$. Assume the variable represents a positive real number.

Simplify. Assume all variables represent positive real numbers.

103. $\sqrt{200}$

104. $\sqrt[3]{16}$

105. $\sqrt[4]{1250}$

106. $-\sqrt{\frac{16}{3}}$

107. $-\sqrt[3]{\frac{2}{5p^2}}$

108. $\sqrt{\frac{2^7y^8}{m^3}}$

109. $\sqrt[4]{\sqrt[3]{m}}$

110. $\frac{\sqrt[4]{8p^2q^5}\cdot\sqrt[4]{2p^3q}}{\sqrt[4]{p^5q^2}}$

111. $\left(\sqrt[3]{2}+4\right)\left(\sqrt[3]{2^2}-4\sqrt[3]{2}+16\right)$

112. $\frac{3}{\sqrt{5}}-\frac{2}{\sqrt{45}}+\frac{6}{\sqrt{80}}$

113. $\sqrt{18m^3}-3m\sqrt{32m}+5\sqrt{m^3}$

114. $\frac{2}{7-\sqrt{3}}$

115. $\frac{6}{3-\sqrt{2}}$

116. $\frac{k}{\sqrt{k}-3}$

Concept Check *Correct each INCORRECT statement by changing the right-hand side of the equation.*

117. $x(x^2+5)=x^3+5$

118. $-3^2=9$

119. $(m^2)^3 = m^5$

120. $(3x)(3y) = 3xy$

121. $\dfrac{\left(\dfrac{a}{b}\right)}{2} = \dfrac{2a}{b}$

122. $\dfrac{m}{r} \cdot \dfrac{n}{r} = \dfrac{mn}{r}$

123. $\dfrac{1}{(-2)^3} = 2^{-3}$

124. $(-5)^2 = -5^2$

125. $\left(\dfrac{8}{7} + \dfrac{a}{b}\right)^{-1} = \dfrac{7}{8} + \dfrac{b}{a}$