## TOWNSHIP OF UNION PUBLIC SCHOOLS



Grade 8 Algebra I
Adopted June 20, 2017
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## Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

## Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

## Course Description

Algebra I provides a formal development of the algebraic skills and concepts necessary for students to succeed in advanced courses. In particular, the instructional program in this course provides for the use of algebraic skills in a wide range of problem-solving situations. The concept of function is emphasized throughout the course. Topics include: (1) operations with real numbers, (2) linear equations and inequalities, (3) relations and functions, (4) polynomials, (5) algebraic fractions, and (6) nonlinear equations.

## Recommended Textbooks:

## Eureka Math - EngageNY Algebra I

Unit 1: Modeling with Linear Equations and Inequalities
Unit 2: Modeling with Linear Functions, Linear Systems, \& Exponential Functions
Unit 3: Quadratic Equations, Functions \& Polynomials
Unit 4: Modeling with Statistics
I. Unit Standards Overview

| Overview | Standards for Mathematical Content | Unit Focus | Standards for Mathematical Practice |
| :---: | :---: | :---: | :---: |
| Unit 1 <br> Modeling with <br> Linear <br> Equations and Inequalities |  | - Reason quantitatively and use units to solve problems <br> - Solve [linear] equations and inequalities in one variable <br> - Understand solving equations as a process of reasoning and explain the reasoning <br> - Create equations that describe numbers or relationships <br> - Interpret the structure of expressions <br> - Represent and solve equations graphically <br> - Summarize, represent, and interpret data on quantitative variables. Interpret linear models | MP. 1 Make sense of problems and persevere in solving them. |
| Unit 1: <br> Suggested Educational Resources | N.Q.A. 1 Runners' World <br> N.Q.A. 2 Giving Raises <br> N.Q.A. 3 Calories in a Sports Drink A.REI.B.3, A.REI.A. 1 Reasoning with inequalities <br> A.CED.A. 4 Equations and Formulas | A.SSE.A. 1 Kitchen Floor Tiles A.CED.A. 1 Planes and wheat A-CED.A. 1 Paying the rent A.REI.A. 1 Zero Product Property 1 A.CED.A. 2 Clea on an Escalator S.ID.B.6.S.ID.C.7-9 Coffee and Crime | MP. 2 Reason abstractly and quantitatively. <br> MP. 3 Construct viable arguments \& critique the reasoning of others. |
| Unit 2 <br> Modeling with <br> Linear <br> Functions, Linear Systems, <br> \& Exponential Functions | (©) A.REI.C.6 $\square$ F.BF.A.1 <br>  A.CED.A.3   <br> ©    <br> A.REI.C.5 $\square$ A.SSE.A.1  <br> A.SSE.B.3    <br> A.REI.D.12 $\square$ F.IF.B.4  <br> F.IF.A.1 $\square$ F.LE.B.5  <br> $\square$ F.IF.A.2 $\square$ F.IF.B.5 <br> $\square$ F.LE.A.1 $\square$ F.IF.B.6 <br> $\square$ F.LE.A.2 $\square$ F.IF.C. 9 <br>  F.IF.A.3 $\square$ F.IF.C. | - Solve linear systems of equations <br> - Create equations that describe numbers or relationships <br> - Interpret the structure of expressions <br> - Represent and solve equations and inequalities graphically <br> - Construct \& compare linear \& exponential models <br> - Interpret expressions for functions in terms of the situation <br> - Build a function that models a relationship between two quantities <br> - Understand the concept of a function and use function notation <br> - Interpret functions that arise in applications in terms of the context <br> - Analyze functions using different representations | MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 7 Look for and make use of structure. |
| Unit 2: <br> Suggested Educational Resources |  |  | MP. 8 Look for and express regularity in repeated reasoning. |



## II. Units

## Unit 1

| Unit 1 Modeling with Linear Equations and Inequalities |  |  |  |
| :---: | :---: | :---: | :---: |
| Content \& Practice Standards | SMP | Critical Knowledge \& Skills | Standard Mastery Examples <br> Can be used on formative, summative, benchmark, and alternative assessments. |
| N.Q.A.1. Use units as a way to understand problems and to guide the solution of multistep problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays. <br> N.Q.A.2. Define appropriate quantities for the purpose of descriptive modeling. <br> N.Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. <br> MP. 5 Use appropriate tools strategically. | Concept(s): <br> - Units are associated with variables in expressions and equations in context. <br> - Quantities may be used to model attributes of real world situations. <br> - Measurement tools have an inherent amount of uncertainty in measurement. <br> Students are able to: <br> - use units to understand real world problems. <br> - use units to guide the solution of multi-step real world problems (e.g. dimensional analysis). <br> - choose and interpret units while using formulas to solve problems. <br> - identify and define appropriate quantities for descriptive modeling. <br> - choose a level of accuracy when reporting measurement quantities. <br> Learning Goal 1: Solve multi-step problems, using units to guide the solution, interpreting units consistently in formulas and choosing an appropriate level of accuracy on measurement quantities. Develop descriptive models by defining appropriate quantities. | When a pitcher throws a 90 mph fastball, how soon does the pitch arrive at home plate? $[90 \mathrm{mph}=132 \mathrm{ft} / \mathrm{sec}]$ <br> Steve wants to add built-in bookshelves along a certain wall in his home. The wall is 10 feet tall and he plans to install 8 evenly spaced 1 inch thick shelves. [Estimate or Exact Solution Necessary?] |

$\square$ A.REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

- A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- A.CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R .
- A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context.
A.SSE.A.1a. Interpret parts of an expression, such as
terms, factors, and
coefficients.
A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.
A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from

MP 2 Reason abstractly and $\quad$ Concept(s). quantitatively.

MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.
MP. 1 Make sense of
problems and persevere in
solving them.

MP 2 Reason abstractly and quantitatively.

MP 2 Reason abstractly and quantitatively.
MP. 4 Model with mathematics.

MP. 7 Look for and make use of structure. process.

Concept(s):

- Literal equations can be rearranged using the properties of equality.
Students are able to.
- solve linear equations with coefficients represented by letters in one variable.
- use the properties of equality to justify steps in solving linear equations.
- solve linear inequalities in one variable.
- rearrange linear formulas and literal equations, isolating a specific variable.

Learning Goal 2. Solve linear equations and inequalities in one variable (including literal equations); justify each step in the

## Concept(s): No new concept(s) introduced

Students are able to:

- identify different parts of an expression, including terms, factors and constants
- explain the meaning of parts of an expression in context.

Learning Goal 3: Interpret terms, factors, coefficients, and other parts of expressions in terms of a context.

- Equations and inequalities describe relationships.
- Equations can represent real-world and mathematical problems.
Students are able to:
- identify and describe relationships between quantities in word problems.
- create linear equations in one variable.
- create linear inequalities in one variable.
- use equations and inequalities to solve real world

Rearrange Ohm's law V = IR to highlight resistance R (literal equations).

Use deductive reasoning and properties of equality to show that $\frac{3(2 x+8)}{6}-4=x$. Justify each step.

Solve $6 x-15<4 x+11$.

Give an example of two like terms and two unlike terms. Explain why they would or would not be classified as like terms.

A music store sells a copy of an electric guitar for $\$ 295$. This is $\$ 30$ more than a third of the cost of the electric guitar it is modeled after. What is the cost of the electric guitar?

In order to get a bonus this month, Leon must sell at least 120 newspaper subscriptions. He sold 85 subscriptions in the first three weeks of the month. How many subscriptions must Leon sell in the

| the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |  | problems. <br> - explain each step in the solution process. <br> Learning Goal 4: Create linear equations and inequalities in one variable and use them in contextual situations to solve problems. Justify each step in the process and the solution. | last week of the month? |
| :---: | :---: | :---: | :---: |
| - A.CED.A.2. Create equations in two or more variables to represent relationships between quantities; Graph equations on coordinate axes with labels and scales. N.Q.A.1. Use units as a way to understand problems and to guide the solution of multistep problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays. <br> A.REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Focus on linear equations.] | MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Equations represent quantitative relationships. <br> Students are able to: <br> - create linear equations in two variables, including those from a context. <br> - select appropriate scales for constructing a graph. <br> - interpret the origin in graphs. <br> - graph equations on coordinate axes, including labels and scales. <br> - identify and describe the solutions in the graph of an equation. <br> Learning Goal 5: Create linear equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | A carpenter buys finishing nails by the pound. Each pound of nails cost $\$ 1.19$. Write a function rule to describe this relationship. <br> Which of the following statements is NOT true of the origin? <br> a. The origin is at the point $(0,0)$. <br> b. The origin is where the x -axis and y -axis intersect. <br> c. The origin is where $x=0$ and $y=$ 0 . <br> d. The origin is where x is greater than y . |
| S.ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.B.6a. Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. <br> MP. 5 Use appropriate tools strategically. | Concept(s): <br> - Scatter plots represent the relationship between two variables. <br> - Scatter plots can be used to determine the nature of the association between the variables. <br> - Linear models may be developed by fitting a linear function to approximately linear data. <br> - The correlation coefficient represents the strength of a linear association. <br> Students are able to: | Make a scatter plot of the given data on the age of a car and the asking price of several used mid-sized cars. Using the data to find the line of best fit and the associated function. Answer specific questions by way of extrapolation and interpolation of data. [Incorporate Graphing Calculator] <br> The graph shows the altitude of an airplane as it comes in for a landing. In comparing |

context. Emphasize linear, quadratic, and exponential models.
S.ID.B.6c. Fit a linear function for a scatter plot that suggests a linear association.

- S.ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
$\square$ S.ID.C.8. Compute (using technology) and interpret the correlation coefficient of a
linear fit.
- S.ID.C.9. Distinguish between correlation and causation.
A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=$ $g(x)$ intersect are the solutions of the equation $\mathrm{f}(\mathrm{x})=\mathrm{g}(\mathrm{x})$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [Focus on linear equations.]

MP. 6 Attend to precision

MP. 1 Make sense of problems and persevere in solving them.

MP. 3 Construct viable arguments and critique the reasoning of others.

MP. 5 Use appropriate tools strategically.

- distinguish linear models representing approximately linear data from linear. equations representing "perfectly" linear relationships.
- create a scatter plot and sketch a line of best fit.
- fit a linear function to data using technology.
- solve problems using prediction equations.
- interpret the slope and the intercepts of the linear model in context.
- determine the correlation coefficient for the linear model using technology.
- determine the direction and strength of the linear association between two variables.

Learning Goal 6: Represent data on a scatter plot, describe how the variables are related and use technology to fit a function to data.
Learning Goal 7: Interpret the slope, intercept, and correlation coefficient of a data set of a linear model; distinguish between correlation and causation.

## Concept(s):

- $y=f(x), y=g(x)$ represent a system of equations.
- Systems of equations can be solved graphically (8.EE.C.8).

Students are able to:

- explain the relationship between the x-coordinate of a point of intersection and the solution to the equation $f(x)$ $=g(x)$ for linear equations $y=f(x)$ and $y=g(x)$.
- find approximate solutions to the system by making a table of values, graphing, and finding successive approximations.

Learning Goal 8: Explain why the solutions of the equation $f(x)=$ $\mathrm{g}(\mathrm{x})$ are the x -coordinates of the points where the graphs of the linear equations $\mathrm{y}=\mathrm{f}(\mathrm{x})$ and $\mathrm{y}=\mathrm{g}(\mathrm{x})$ intersect. ** function notation is not
the time to the altitude, find the rate of change and distinguish the correlation.

|  |  | introduced here |  |
| :--- | :--- | :--- | :--- |
|  |  | Learning Goal 9: Find approximate solutions of $\mathrm{f}(\mathrm{x})=\mathrm{g}(\mathrm{x})$, <br> where $\mathrm{f}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x})$ are linear functions, by <br> making a table of values, using technology to <br> graph and finding successive approximations. |  |

## Unit 1 Vocabulary

Variable, algebraic expression, equation, evaluate, simplify, exponent, base, power, rational number, irrational number, real numbers, inequality, opposites, absolute value, coordinate plane, coordinates, ordered pair, function, function rule, domain, range, dependent variable, independent variable, scatter plot, correlation, line of best fit, measures of central tendency, additive inverse, matrix, multiplicative inverse, reciprocal, term, coefficient, equivalent equations, solution, consecutive integers, equivalent inequalities, relation, vertical-line test, function notation, continuous data, discrete data, direct variation, inverse variation, inductive reasoning, conjecture, rate of change, slope, linear function, linear equation, slopeintercept form, standard form, y -intercept, x -intercept, point-slope form, parallel and perpendicular lines.

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 1 Suggested Open Educational Resources
b. Two quizzes/Two Tests tests
c. Desmos www.desmos.com
d. Scavenger Hunt
e. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
a. Use of Higher Level Questioning Techniques
b. Extension/Challenge Questions
c. Provide Assessments at a Higher Level of Thinking
d. Desmos Linear Activities - https://teacher.desmos.com/linear

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- Using a computer model or graphing calculator, graph multiple linear equations that vary in slope but have the same y-intercept. How is slope represented on the graph of a linear equation?


## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.

### 9.2 Career A wareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
9.2.8.B. 4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.
9.2.8.B.5 Analyze labor market trends using state and federal labor market information and other resources available online.
9.2.8.B. 7 Evaluate the impact of online activities and social media on employer decisions.

## Career \& Technical Education Content Area: $21{ }^{\text {st }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

Interdisciplinary Connections/Global Perspective: Research the different units of measure used around the world. Write an equation to convert each of the different units of measure used for length into the customary system used in the United States. Describe some of the benefits or drawbacks of the different systems of measurement.

## Unit 2

Unit 2 Modeling with Linear Functions, Linear Systems, \& Exponential Functions

| Content \& Practice Standards | SMP | Critical Knowledge \& Skills | Standard Mastery Examples <br> Can be used on formative, summative, benchmark, and alternative assessments. |
| :---: | :---: | :---: | :---: |
| © A.REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. <br> A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. <br> © A.REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 3 Construct viable arguments and critique the reasoning of others. MP. 4 Model with mathematics. | Concept(s): <br> - Systems of equations can be solved exactly (algebraically) and approximately (graphically). <br> Students are able to: <br> - identify and define variables representing essential features for the model. <br> - model real world situations by creating a system of linear equations. <br> - solve systems of linear equations using the elimination or substitution method. <br> - solve systems of linear equations by graphing. <br> - interpret the solution(s) in context. <br> Learning Goal 1: Solve multistep contextual problems by identifying variables, writing equations, and solving systems of linear equations in two variables algebraically and graphically. | Fishing Adventures rents small fishing boats to tourists for day long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 1200 pounds of people and gear for safety reasons. Assume on average an adult weighs 150 pounds and a child weighs 75 pounds. Also assume each group will require 200 pounds of gear plus 10 pounds of gear per person. <br> a. Write an inequality that illustrates the weight limit for a group of adults and children on the fishing boat and a second inequality that represents the total number of passengers in the fishing boat. |
| - A.REI.D.12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - model real world situations by creating a system of linear inequalities given a context. <br> - interpret the solution(s) in context. <br> Learning Goal 2: Graph linear inequalities and systems of | Fishing Adventures rents small fishing boats to tourists for day long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 1200 pounds of people and gear for safety reasons. Assume on average an adult weighs 150 pounds and a child weighs 75 pounds. Also assume each group will require 200 pounds of gear plus 10 pounds of gear per person. |

$\square$ A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
$\square$ F.IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of $f$ is the graph of the equation $\mathrm{y}=\mathrm{f}(\mathrm{x})$.
F.IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MP. 5 Use appropriate tools strategically.

MP. 6 Attend to precision.
linear inequalities in two variables and explain that the solution to the system.
a. Write an inequality that illustrates the weight limit for a group of adults and children on the fishing boat and a second inequality that represents the total number of passengers in the fishing boat.
b. Interpret the solutions.

MP 2 Reason abstractly and $\quad$ Concept(s): quantitatively.

MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.

Students are able to:

- $F(x)$ is an element in the range and $x$ is an element in the domain.
- use the definition of a function to determine whether a relationship is a function.
- use function notation once a relation is determined to be a function.
- evaluate functions for given inputs in the domain.
- explain statements involving function notation in the context of the problem.

Jerome is constructing a table of values that satisfies the definition of a function.

\section*{| Input | -13 | 20 | 0 | -4 | 11 | -1 | 17 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output | -15 | -11 | -9 | -2 | -1 | 5 | 5 | 13 |} What number(s) can be placed in the empty cell so that the table of values satisfie the definition of a function? Select all that apply

$\begin{array}{llllll}\text { A. }-5 & \text { B. }-1 & \text { C. } 0 & \text { D. } 2 & \text { E. } 11 & \text { F. } 17\end{array}$

|  | F.LE.A.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. <br> F.LE.A.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. <br> F.LE.A.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. <br> F.LE.A.1c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| :---: | :---: |
|  | F.LE.A.2. Construct linear and exponential functions including arithmetic and geometric sequences - given a graph, a description of a relationship, or two inputoutput pairs (include reading these from a table). <br> *[Algebra 1 limitation: exponential expressions with integer exponents] <br> F.IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined |

MP. 3 Construct viable arguments and critique the reasoning of others. MP. 6 Attend to precision.

## MP 2 Reason abstractly and Concept(s):

 quantitatively.MP 4. Model with mathematics

MP. 1 Make sense of problems and persevere in solving them.

MP. 5 Use appropriate tools strategically.

MP. 6 Attend to precision.
MP. 7 Look for and make use

Concept(s):

- Linear functions grow by equal differences over equal intervals.
- Exponential functions grow by equal factors over equal intervals.
Students are able to:
- identify and describe situations in which one quantity changes at a constant rate.
- identify and describe situations in which a quantity grows or decays by a constant percent.
- show that linear functions grow by equal differences over equal intervals.
- show that exponential functions grow by equal factors over equal intervals.

Learning Goal 4: Distinguish between and explain situations modeled with linear functions and with exponential functions.

Determine whether the following ordered pairs represent a linear or an exponential relationship.

An initial population of 5 squirrels increases by $9 \%$ each year for 10 years. Using x for years and y for the number of squirrels, write the equation that models this situation. How many squirrels will there be in 10 years?

A car purchased for $\$ 34,000$ is expected to lose value, or depreciate, at a rate of $6 \%$ per year. Using $x$ for years and $y$ for the value of the car, write the equation that models this situation. After how many years is the car first worth less than $\$ 21,500$ ?

Write a geometric sequence. You must include the first four terms of your sequence. Identify your common ratio then write an equation to represent the rule.

| recursively by $f(0)=f(1)=1$, $f(n+1)=f(n)+f(n-1)$ for $n \geq$ 1. | of structure. | - a graph; <br> - a description of a relationship; <br> - a table of values. <br> Learning Goal 5: Write linear and exponential functions given a graph, table of values, or written description; construct arithmetic and geometric sequences. |  |
| :---: | :---: | :---: | :---: |
| F.BF.A.1. Write a function that describes a relationship between two quantities. <br> 1a. Determine an explicit expression, a recursive process, or steps for calculation from a context. <br> A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context <br> A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients. <br> A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$. <br> *[Algebra 1 limitation: exponential expressions with integer exponents] | MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - given a context, write an explicit expressions, a recursive process or steps for calculation for linear and exponential relationships. <br> - interpret parts of linear and exponential functions in context. <br> Learning Goal 6: Write explicit expressions, recursive processes and steps for calculation from a context that describes a linear or exponential relationship between two quantities. | All exponential functions are in the form $y=a(b)^{x}$. <br> What values of $b$ make it an exponential growth function? <br> What values of $b$ make it an exponential decay function? |
| A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - use the properties of exponents to simplify or | Simplify. $\left(a^{-2} b^{3}\right)^{-2}\left(b^{3} c^{-4}\right)^{2}$ |

expression.
A.SSE.B.3c. Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^{t}$ can be rewritten as $\left(1.15^{1 / 12}\right)^{12 t} \approx 1.012^{12 t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$.
*[Algebra 1: limit to exponential expressions
with integer exponents]
F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *[Focus on exponential functions]F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.
F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in
quantitatively.
MP. 4 Model with mathematics.

MP. 7 Look for and make use of structure

MP 2 Reason abstractly and quantitatively.

MP. 4 Model with mathematics.

MP. 6 Attend to precision.
expand exponential expressions, recognizing these are equivalent forms.

Learning Goal 7: Use properties of exponents to produce equivalent forms of exponential expressions in one variable.

Concept(s): No new concept(s) introduced
Students are able to:

- given a verbal description of a relationship, sketch linear and exponential functions.
- identify intercepts and intervals where the function is positive/negative.
- interpret parameters in context.
- determine the practical domain of a function.

Learning Goal 8: Sketch graphs of linear and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context.

$\left(\frac{1}{3 a^{3} b^{2}}\right)^{-4} \cdot\left(-3 a^{10} b^{9}\right)^{-1}$

Examine the graphs of $f(x)=3^{x}$ and $g(x)=5 x$, shown below.

a. Estimate the values of $x$ for which $f(x)$ is greater than $g(x)$
b. Estimate the values of $x$ for which $g(x)$ is greater than $f(x)$.

| a factory, then the positive integers would be an appropriate domain for the function |  |  |  |
| :---: | :---: | :---: | :---: |
| $\square$ F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <br> For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. <br> *[Limit to linear and exponential] <br> - F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 8 Look for and express regularity in repeated reasoning. | Concept(s): <br> - Rate of change of non-linear functions varies. Students are able to: <br> - compare key features of two linear functions represented in different ways. <br> - compare key features of two exponential functions represented in different ways. <br> - calculate the rate of change from a table of values or from a function presented symbolically. <br> - estimate the rate of change from a graph. <br> Learning Goal 9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <br> Learning Goal 10: Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph. | In 2007, Zack bought a new car for $\$ 17,500$. The table below shows the value of the car between 2007 and 2012. <br> Part A. Calculate the average rate of change of the value of the car between 2007 and 2008. Explain what your answer means in terms of the car's value over this interval. <br> Part B. Calculate the average rate of change of the value of the car between 2008 and 2012. Explain what your answer means in terms of the car's value over this interval. <br> Part C. Compare the values from Part A and Part B. What can you conclude based on this comparison along with the data in the table in terms of the car's value over the time period shown in the table? Use words, numbers and/or pictures to show your work. |
| F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 5 Use appropriate tools | Concept(s): <br> - Piecewise-defined functions may contain discontinuities. <br> - Absolute value functions are piecewise functions. | Graph $f(x)=\|x\|$ and the resulting graphs of the expanded functions. Compare and contrast the behavior of these graphs to $f(x)=x$ and its expanded versions $f(x)=a x+c$. |

for more complicated cases.
F.IF.C.7b. Graph
piecewise-defined
functions, including step functions and absolute value functions
strategically.
MP. 6 Attend to precision.

## Students are able to:

- graph linear, piecewise-defined functions.
- graph more complicated cases of functions using technology.
- identify and describe key features of the graphs of piecewise-defined functions .

Learning Goal 11: Graph linear and piecewise-defined functions (including step and absolute value functions) expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph.

## Unit 2 Vocabulary

System, elimination, linear combinations, substitution, consistent and inconsistent system, dependent and independent system, infinitely many solutions, no solution, half-plane, exponent, negative exponent power, base, order of magnitude, Power of a Product/Quotient Property, Product/Quotient of a Power Property, Power of a Power Property, reciprocal, scientific notation, exponential function, exponential growth, exponential decay, compound interest, initial amount, growth/decay factor, growth/decay rate, time.

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 2 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos www.desmos.com
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
e. Use of Higher Level Questioning Techniques
f. Extension/Challenge Questions
g. Provide Assessments at a Higher Level of Thinking
h. Desmos Linear Activities - https://teacher.desmos.com/linear
i. Desmos Exponential Activities - https://teacher.desmos.com/exponential

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- Using a computer model or graphing calculator, graph linear, quadratic, and exponential functions and compare and contrast their characteristics.


## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.

### 9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
9.2.8.B.4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.
9.2.8.B.5 Analyze labor market trends using state and federal labor market information and other resources available online.
9.2.8.B. 7 Evaluate the impact of online activities and social media on employer decisions.

## Career \& Technical Education Content Area: $\mathbf{2 1}^{\text {st }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

## Interdisciplinary Connections:

- Science: Use projectile motion equations to model the height of objects when affected by gravity.
- Social Studies: Write functions to determine rate of return on stock investments.

Media Literacy: Look through the local newspaper or magazine and find 3 advertisements for local or national banks. Compare the advertised rates for checking or savings accounts. Use an exponential equation to compare the different offers and determine which will lead to the greatest interest after 1,5 , and 10 years.

## Unit 3

## Unit 3 Quadratic Equations, Functions \& Polynomials

| Content \& Practice Standards | SMP | Critical Knowledge \& Skills | Standard Mastery Examples <br> Can be used on formative, summative, benchmark, and alternative assessments. |
| :---: | :---: | :---: | :---: |
| A.APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as ( $x^{2}$ $\left.y^{2}\right)\left(x^{2}+y^{2}\right)$. | MP. 2 Reason abstractly and quantitatively. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Polynomials form a system analogous to the integers. <br> - Polynomials are closed under the operations of addition, subtraction, and multiplication. <br> Students are able to: <br> - add and subtract polynomials. <br> - multiply polynomials. <br> - recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences. <br> - recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences. <br> Learning Goal 1: Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable. | Adding polynomials $\left(3 x^{3}-5 x^{2}-7\right)+\left(4 x^{2}-2 x+3\right)$ <br> For illustration we will write this addition vertically: $3 x^{3}-5 x^{2} \quad-7$ <br> $4 x^{2}-2 x+3 \quad$ Line up like terms. $3 x^{3}-x^{2}-2 x-4 \quad \text { Add. }$ <br> Michelle borrowed $3 r^{3}+5 r^{2}+18 r+20$ dollars from her brother. If she paid back $3 r^{3}+2 r^{2}-2 r+11$ dollars, then how much more money does she still owe her brother? <br> Simplify. $(4 b-3)^{2}$ |
| A.REI.B.4. Solve quadratic equations in one variable. A.REI.B.4a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form ( $x$ $p)^{2}=q$ that has the same solutions. Derive the | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools | Concept(s): <br> - Multiple methods for solving quadratic equations. <br> - Transforming a quadratic equation into the form $(x$ $p)^{2}=q$ yields an equation having the same solutions. <br> Students are able to: <br> - use the method of completing the square to transform a quadratic equation in $x$ into an equation of the form | Which method would you choose to solve the equation $3 x^{2}-27=0$ ? Justify your reasoning and solve. |

quadratic formula from this form.
A.REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b$.
strategically.
MP. 7 Look for and make use of structure.
A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.

## MP 2 Reason abstractly

 and quantitatively.MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.
$(x-p)^{2}=q$.

- derive the quadratic formula from $(x-p)^{2}=q$.
- solve a quadratic equations in one variable by inspection.
- solve quadratic equations in one variable by taking square roots.
- solve a quadratic equations in one variable by completing the square.
- solve a quadratic equations in one variable using the quadratic formula.
- solve a quadratic equations in one variable by factoring.
- strategically select, as appropriate to the initial form of the equation, a method for solving a quadratic equation in one variable.
- analyze the quadratic formula, recognizing the conditions leading to complex solutions (discriminant).

Learning Goal 2: Derive the quadratic formula by completing the square and recognize when there are no real solutions.

Learning Goal 3: Solve quadratic equations in one variable
using a variety of methods (including inspection, taking square roots, factoring, completing the square, and the quadratic formula) and write complex solutions in $a \pm b i$ form.

## Concept(s): No new concept(s) introduced

Students are able to:

- create quadratic equations in one variable.
- use quadratic equations to solve real world problems.

Solve $3 \mathrm{x}=2 \mathrm{x}^{2}-2$ using the Quadratic Formula.

You toss a ball that travels on the path $\mathrm{y}=-0.1 \mathrm{x}^{2}+$ $x+2$ where $x$ and $y$ are measured in meters. Sketch the path of the ball. What is the maximum height of the ball? Identify the domain and range of the function.

|  |  | and use them to solve problems. |  |
| :---: | :---: | :---: | :---: |
| $\square$ F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. <br> - F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <br> For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function <br> F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> F.IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. | MP. 4 Model with mathematics. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem. <br> - sketch graphs of quadratic functions given a verbal description of the relationship between the quantities. <br> - identify intercepts and intervals where function is increasing/decreasing <br> - determine the practical domain of a function. <br> Learning Goal 5: Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its graph. |  |


| A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. <br> A.SSE.B.3a. Factor a quadratic expression to reveal the zeros of the function it defines. <br> A.SSE.B.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Alternate, equivalent forms of a quadratic expression may reveal specific attributes of the function that it defines. <br> Students are able to: <br> - factor a quadratic expression for the purpose of revealing the zeros of a function. <br> - complete the square for the purpose of revealing the maximum or minimum of a function. <br> Learning Goal 6: Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlight particular properties such as the zeros or the maximum or minimum value of the function. | Find the zeros of the function $f(x)=x^{2}+2 x-8$. <br> Solve $x^{2}+6 x-7=0$ by completing the square. |
| :---: | :---: | :---: | :---: |
| F.BF.A.1. Write a function that describes a relationship between two quantities. <br> F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context. | MP. 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - given a context, write explicit expressions, a recursive process or steps for calculation for quadratic relationships. <br> Learning Goal 7: Given a context, write an explicit expression, a recursive process or steps for calculation for quadratic relationships. | Write a polynomial expression with integral coefficients that has the given roots: 0 and -2 . |
| F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> F.IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *[emphasize | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - graph quadratic functions expressed symbolically. <br> - graph more complicated cases of quadratic functions using technology. <br> - identify and describe key features of the graphs of quadratic functions. | When the solutions to each of the two equations shown are graphed in the $x-y$ coordinate plane, the graphs of the solutions intersect at a point. What is the y coordinate of the point of intersection? $\begin{gathered} y=x^{2}-2 x-5 \\ y=x^{3}-2 x^{2}-5 x-9 \end{gathered}$ |

$\square$ F.IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.C.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
$\square$ F.LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
strategically.
MP. 6 Attend to precision.
MP. 8 Look for and express regularity in repeated reasoning.


## MP. 1 Make sense of $\quad$ Concept(s):

problems and persevere in solving them.
MP. 4 Model with mathematics.

MP. 5 Use appropriate tools strategically.

MP. 7 Look for and make use of structure. Students are able to:

- given two quadratic functions, each represented in a different way, compare the properties of the functions.

Learning Goal 8: Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.

- A quantity increasing exponentially eventually exceeds a quantity increasing quadratically.
- calculate the rate of change of a quadratic function from a table of values or from a function presented symbolically.
- estimate the rate of change from a graph of a quadratic function.
- analyze graphs and tables to compare rates of change of exponential and quadratic functions.

Graph the quadratic functions $y=-2 x^{2}$ and $y=-2 x^{2}$ +4 . Compare the shape and position of the graphs.

|  |  | change of a quadratic function presented symbolically or as a table. Estimate and compare the rates of change from graphs of quadratic and exponential functions. |  |
| :---: | :---: | :---: | :---: |
| (©) F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Vertical and horizontal shifts <br> Students are able to: <br> - perform transformations on graphs of linear and quadratic functions. <br> - identify the effect on the graph of replacing $\mathrm{f}(\mathrm{x})$ by <br> $-f(x)+k ;$ <br> - $k f(x)$; <br> - $f(k x)$; <br> - and $f(x+k)$ for specific values of $k$ (both positive and negative). <br> - identify the effect on the graph of combinations of transformations. <br> - given the graph, find the value of k . <br> - illustrate an explanation of the effects on linear and quadratic graphs using technology. <br> Learning Goal 10: Identify the effects of transformations and combinations of transformations [ $f(x)$ $+k, k f(x), f(k x)$, and $f(x+k)]$ on a function; find the value of k given the graph. | Make a table of values and graph. Then EXPLAIN how the graph is similar or different to the parent function, $y=x^{2}$. <br> Melissa graphed the equation $y=x^{2}$ and Dave graphed the equation $y=-3 x^{2}$ on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew? <br> Determine graphically using possible symmetry, whether the following functions are even, odd, or neither. |
| A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=$ $g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 5 Use appropriate tools strategically. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - approximate the solution(x) to a system of equations comprised of a linear and a quadratic function by using technology to graph the functions, by making a table of values and/or by finding successive approximations. | The figure shows a graph of the function $f(x)$ in the $x y$-coordinate plane. <br> A second function $g$ is defined by $g(x)=-3 x+2$. Select the correct phrase in each drop-down menu to complete the sentence. $f(2) \left\lvert\,$$\begin{array}{l}\text { Choose ... } \\ \text { is less than } \\ \text { is greater than } \\ \text { is equal to }\end{array}$$\quad g(2)\right.$ and $f(-2)\left\|\begin{array}{l}\text { Choose ... } \\ \text { is less than } \\ \text { is greater than } \\ \text { is equal to }\end{array}\right\| g(-2)$. |


| where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* |  | Learning Goal 11: Find approximate solutions of $f(x)=g(x)$, where $f(x)$ is a linear function and $g(x)$ is a quadratic function by making a table of values, using technology to graph and finding successive approximations. |  |
| :---: | :---: | :---: | :---: |
| A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available] | MP. 7 Look for and make use of structure. | Concept(s): <br> - General shape(s) and end behavior of cubic functions <br> Students are able to: <br> - find the zeros of a polynomial (quadratic and cubic). <br> - understand domain intervals to determine where $f(x)$ is greater than or less than zero. <br> - use zeros of a function to sketch a graph. <br> Learning Goal 12: Identify zeros of cubic functions when suitable factorizations are available and use the zeros to construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor) | Which ordered pairs represent the zeros of the function? $f(x)=\left(x^{2}+2 x-8\right)(x-6)$ <br> Select ALL that apply. <br> a. $(2,0)$ <br> b. $(6,0)$ <br> c. $(0,-8)$ <br> d. $(-4,0)$ <br> e. $(-6,0)$ <br> f. $(0,2)$ <br> g. $(0,8)$ |

## Unit 3 Vocabulary

monomial, polynomial, degree, leading coefficient, binomial, trinomial, roots, vertical motion model, zero of a function, perfect square trinomial, factor by grouping, factor completely, quadratic function, parabola, parent quadratic function, axis of symmetry, vertex, minimum and maximum value, intercept form, quadratic equation, square root, perfect square, completing the square, vertex form, quadratic formula, discriminant, radical expression, radical function, radical equations, rational and irrational numbers.

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 3 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos www.desmos.com
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
j. Use of Higher Level Questioning Techniques
k. Extension/Challenge Questions
5. Provide Assessments at a Higher Level of Thinking
m. Desmos Quadratic Activities - https://teacher.desmos.com/quadratic

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- On a graphing calculator, sketch the graph of $y=x^{2}-3$. Zoom once to get a better view of the positive $x$-intercept. Zoom a second time to get an even better view. Use the cursor keys to determine the $x$-intercept. Zoom a third time. Use the trace feature to find the $y$-value that is closest to 0 . The corresponding x -value is your approximation.


## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.

CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.

### 9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
9.2.8.B.4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.
9.2.8.B. 5 Analyze labor market trends using state and federal labor market information and other resources available online.
9.2.8.B. 7 Evaluate the impact of online activities and social media on employer decisions.

## Career \& Technical Education Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

## Interdisciplinary Connections:

From 1960 to 1990, the total government payroll in the US can be modeled by $P=35 t^{2}+115 t+3410$, where $P$ is in millions of dollars and $t=0$ corresponds to 1960. Judging from the payroll between 1960 and 1990, is it possible that the government payroll will reach 70 billion dollars a year?

## Global Perspective:

The Arecibo Observatory is a radio telescope located in Puerto Rico. The observatory's $1,000 \mathrm{ft}$ radio telescope is the world's largest single-aperture telescope. The shape of the dish used can be modeled with a quadratic equation. Research the dimensions of the telescope and write an equation that closely models the telescope and describe some of the research done by radio telescopes.

## Unit 4

| Unit 4 Modeling with Statistics |  |  |  |
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| Content \& Practice Standards | SMP | Critical Knowledge \& Skills | Standard Mastery Examples <br> Can be used on formative, summative, benchmark, and alternative assessments. |
| F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. <br> F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <br> For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. | MP. 4 Model with mathematics. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - interpret maximum/minimum and intercepts of functions from graphs and tables in the context of the problem. <br> - sketch graphs of functions given a verbal description of the relationship between the quantities. <br> - identify intercepts and intervals where function is increasing/decreasing. <br> - determine the practical domain of a function . <br> Learning Goal 5: Interpret key features of functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a function, showing key features and relating the domain of the function to its graph. | Jamie throws a ball that will move through the air in a parabolic path due to gravity. The height, h , in meters, of the ball above the ground after $t$ seconds can be modelled by the function $h(t)=-$ $4.9 t^{2}+40 t+1.5$. <br> a) Sketch the graph of the function. <br> b) Find the zeros of the function and interpret their meaning. <br> c) Determine the time needed for the ball to reach its maximum height. <br> d) What is the maximum height of the ball? <br> Describe the graph that would represent the rate at which different shaped containers are filled with water. <br> Use the graph to find: <br> (a) Its domain and range <br> (b) The $x$ - and $y$ - intercepts <br> (c) The intervals of increase. Justify. <br> (d) The intervals of decrease. Justify. <br> (e) The intervals of constant. Justify. |


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| S.ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.B.6a. Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. <br> S.ID.B.6b. Informally assess the fit of a function with the use of technology. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - fit a function to data using technology. <br> - solve problems using functions fitted to data (prediction equations). <br> - interpret the intercepts of models in context. <br> Learning Goal 4: Fit functions to data using technology.. | Is there a relationship between the fat grams and the total calories in fast food? <br> Use the graphing calculator to predict the total calories based upon 22 grams of fat. |
| F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> F.IF.C.7b. Graph square root and cube root functions. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. | Concept(s): <br> - General understanding of the graphs of square root and cube root functions. <br> Students are able to: <br> - Graph square root and cube root functions. <br> - graph more complicated cases of functions using | Describe the behavior of the graph of $\mathrm{y}=\sqrt{x}$ and $\mathrm{y}=\sqrt[3]{x}$. <br> Describe the behavior of the graph of $y=$ $\sqrt{x} \pm k$ and $\mathrm{y}=\sqrt[3]{\boldsymbol{x}} \pm \boldsymbol{k}$. |


|  |  | technology. <br> - identify and describe key features of the graphs of square root and cube root functions . <br> Learning Goal 6: Square root and cube root functions expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph. |  |
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| (© N.RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 6 Attend to precision. | Concept(s): <br> - The sum or product of two rational numbers is rational. <br> - The sum of a rational number and an irrational number is irrational. <br> - The product of a nonzero rational number and an irrational number is irrational. <br> Students are able to: <br> - explain and justify conclusions regarding sums and products of two rational numbers.. <br> - explain and justify conclusions regarding the sum of a rational and irrational number. <br> - explain and justify conclusions regarding the product of a nonzero rational and irrational number. <br> Learning Goal 7: Explain and justify conclusions about sums and products of rational and irrational numbers. | Indicate whether each statement is true or false. <br> - The sum of two rational numbers is always rational. <br> - The sum of a rational number and an irrational number is sometimes rational. <br> - The product of two rational numbers is sometimes rational. <br> - The product of two irrational numbers is never rational. |
| © S.ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - represent data with dot plots on the real number line. <br> - represent data with histograms on the real number line. <br> - represent data with box plots on the real number line. | A random sample of teenagers ages 13 and 14 were asked: On average, how many text messages do you send per day? Here are the results: $\begin{gathered} 0,10,10,10,10,20,20,20,30,50,50,50,90 \\ 100,100 \end{gathered}$ <br> Display the data in a dot plot. What can you say |


|  | MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. | Learning Goal 1: Represent data with plots (dot plots, histograms, and box plots) on the real number line. | about the results? <br> Let's say you were interested in finding out how many teenagers send 50 or more text messages per day and how many send less than 50 text messages per day. You can count the number of dots in those categories and make a table. Then display the data in a histogram. <br> Michelle looks at the text messaging data and wants to describe the spread of numbers above and below the median. Display the data in a box plot to help Michelle describe the spread of numbers. |
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| (©) S.ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. <br> © S.ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. | Concept(s): <br> - Appropriate use of a statistic depends on the shape of the data distribution. <br> - Standard deviation <br> Students are able to: <br> - represent two or more data sets with plots and use appropriate statistics to compare their center and spread. <br> - interpret differences in shape, center, and spread in context. <br> - explain possible effects of extreme data points (outliers) when summarizing data and interpreting shape, center and spread. <br> Learning Goal 2: Compare center and spread of two or more data sets, interpreting differences in shape, center, and spread in the context of the data, | Consider the following three data sets A, B and C. $\begin{aligned} & A=\{9,10,11,7,13\} \\ & B=\{10,10,10,10,10\} \text { Find } \\ & C=\{1,1,10,19,19\} \end{aligned}$ <br> a) Calculate the mean of each data set. <br> b) Calculate the standard deviation of each data set. <br> c) Which set has the largest standard deviation? <br> d) Is it possible to answer question c) without calculations of the standard deviation? <br> The accompanying box-and-whisker plots can be used to compare the annual incomes of three professions. |


8.G.B.7. Apply the

Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

- 8.G.B.8. Apply the

Pythagorean Theorem to find the distance between two points in a coordinate system.

MP. 2 Reason abstractly and quantitatively.

MP. 7 Look for and make use of structure.

Concept(s): No new concept(s) introduced
Students are able to:

- determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems in two and three dimensions.
- determine the distance between two points in a coordinate plane by applying the Pythagorean Theorem.

Learning Goal 8: Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.

Find the distance between each pair of coordinates. $(-4,-3)$ and $(-8,7)$

Find the midpoint of the line segment with given endpoints: $(-4,9)$ and $(1,2)$

If I walk 3 blocks East and 4 blocks North, how far am I from my starting point?

## Unit 4 Vocabulary

Rational, Irrational, Mean, Median, Mode, Range, Interquartile Range, Box Plot, Dot Plot, Histogram, Two-Way Table, Frequency, Relative Frequency, Conditional Frequency, Joint Frequency, Marginal Frequency, Ratio, Percent, Scale, Outliers, Standard Deviation, Constant, Increase, Decrease, Domain, Range, Maximum, Minimum

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 4 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos www.desmos.com
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
n. Use of Higher Level Questioning Techniques
o. Extension/Challenge Questions

Provide Assessments at a Higher Level of Thinking

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.

### 9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
9.2.8.B.4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.
9.2.8.B. 5 Analyze labor market trends using state and federal labor market information and other resources available online.
9.2.8.B.7 Evaluate the impact of online activities and social media on employer decisions.

## Career \& Technical Education Content Area: $\mathbf{2 1}^{\text {st }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

Interdisciplinary Connections: Science: Develop a scatter plot for an experiment (for example, the relationship between temperature and pressure for a given volume). Identify the trend line and determine if the relationship is linear.

Media Literacy: Voter polls are much more accurate than they used to be. The science of random selection poll participants began after a poll conducted in 1936 inaccurately predicted who would win the Presidential election. Find a news article that uses poll data as a source of information. Research the polling company and the methods used and describe why random selection polling is an accurate way of making predictions.
III. Additional Differentiation/Modifications for Teaching


## IV. Instructional Resources and Materials

| Formative Assessment | Summative Assessment | Supplemental | Resources | Print Resource |
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| Short constructed responses <br> Extended constructed responses <br> Teacher Observation <br> Checks for understanding <br> Do Now <br> Exit Tickets <br> Problem Sets (EngageNY) <br> Sprints (EngageNY) <br> Extension - See additional performance tasks in the Unit Standards Overview. | End-of-Module Assessment (EngageNY) <br> Mid-Module Assessment (EngageNY) | Teacher Resources <br> Annenberg Learning <br> Mathematics Assessment Projects <br> Achieve the Core <br> Mathplanet.com <br> Interactive <br> Mathematics.com <br> Illustrative Mathematics <br> Inside Mathmatics.org <br> EdConnect.org <br> Prodigy <br> Desmos <br> iReady <br> Khan Academy | Student Resources <br> Khan Academy <br> Prodigy <br> iReady <br> Math is Fun (website) <br> Virtual Nerd <br> Engage NY <br> (website) <br> Engage NY <br> (Homework <br> Helpers) <br> A Math <br> Dictionary for Kids | Eureka Math - <br> Engage NY <br> Algebra I <br> Mathematics |

Algebra 1 (Middle School) Pacing Guide



