## TOWNSHIP OF UNION PUBLIC SCHOOLS



UHS Algebra Lab
Approved September 18, 2018 Updated December 18, 2018

## Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

## Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

## Course Description

Algebra Lab is a full-year course divided into two sections. The first half of the year will focus on solving systems of equations, exploring linear, quadratic, exponential, and rational functions, and manipulating radical, polynomial, and rational expressions. The second half of this course focuses on the language of geometry, reasoning and introduction to proof, parallels, congruent triangles and quadrilaterals. Graphing calculators are used throughout this course to further develop each concept.

## Recommended Program:

## APEX Learning

Unit 1: Linear Equations and Inequalities/Linear Functions, Linear Systems, \& Exponential Functions
Unit 2: Quadratic Equations, Functions \& Polynomials/ Modeling with Statistics
Unit 3: Congruence Similarity and Proof
Unit 4: Trigonometric Ratios \& Geometric Equations

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## I. Unit Standards Overview

| Overview | Standards for Mathematical Content | Unit Focus | Standards for Mathematical Practice |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { Unit } 1 \\ & \hline \text { Algebra I } \end{aligned}$ | $\square$ N.Q.A.1 $\quad \square$ A.REI.A.1 | - Reason quantitatively and use units to solve problems <br> - Solve [linear] equations and inequalities in one variable <br> - Understand solving equations as a process of reasoning and explain the reasoning <br> - Create equations that describe numbers or relationships <br> - Interpret the structure of expressions <br> - Represent and solve equations graphically <br> - Summarize, represent, and interpret data on quantitative variables. Interpret linear models <br> - Understand the concept of a function and use function notation | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 2 Reason abstractly and quantitatively. |
| Unit 1: <br> Suggested <br> Educational <br> Resources | N.Q.A. 1 Runners' World <br> N.Q.A. 2 Giving Raises <br> N.Q.A. 3 Calories in a Sports Drink A.REI.B.3, A.REI.A. 1 Reasoning w inequalities <br> A.CED.A. 4 Equations and Formulas |  A.SSE.A. 1 Kitchen Floor Tiles <br>  <br>  <br> A.CED.A. 1 Planes and wheat <br> A.CED.A. 1 Paying the rent <br> A-CEar <br> A.REI.A. Zero Product Property 1 <br> A.CED.A. 2 Clea on an Escalator <br> S.ID.B.6,S.ID.C.7-9 Coffee and Crime  <br>   | MP. 3 Construct viable arguments \& critique the reasoning of others. <br> MP. 4 Model with mathematics. |
| $\frac{\text { Unit } 2}{\text { Algebra I }}$ |  | - Solve linear systems of equations <br> - Create equations that describe numbers or relationships <br> - Interpret the structure of expressions <br> - Represent and solve equations and inequalities graphically <br> - Construct \& compare linear \& exponential models <br> - Interpret expressions for functions in terms of the situation <br> - Build a function that models a relationship between two quantities <br> - Interpret functions that arise in applications in terms of the context <br> - Analyze functions using different representations | MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 7 Look for and make use of structure. |
| Unit 2: <br> Suggested Educational Resources | A.REI.C. 6 Cash Box <br> A.CED.A. 3 Dimes and Quarters <br> A.REI.C. 5 Solving Two Equations i <br> A.REI.D. 12 Fishing Adventures 3 <br> F.IF.A. 1 The Parking Lot <br> F.IF.A. 2 Yam in the Oven <br> F.LE.A. 1 Finding Linear and Expon <br> F.LE.A. 2 Interesting Interest Rates |  F.BF.A.1a Skeleton Tower <br> Two Unknowns <br>  <br> A.SSE.A.1 Mixing Candies <br> F.IF.B.4 Warming and Cooling <br>  F.IF.B.4, F.IF.B.5 Average Cost <br> F.LE.B.5 US Population 1982-1988 <br> Fodels <br> F.IF.B.6 Temperature Change  <br> F.IF.C.7b Bank Account Balance  | MP. 8 Look for and express regularity in repeated reasoning. |

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| Overview | Standards for Mathematical <br> Content$\quad$ Unit Focus | Standards for Mathematical Practice |
| :---: | :---: | :---: |
| Unit 3 <br> Congruence <br> Similarity and Proof | $\square$    <br> G.SRT.A.1 $\square$ G.CO.C.10 • Understand similarity in terms of similarity transformations <br> G.SRT.A. 2 $\square$ G.CO.C.11 • Prove geometric theorems. <br> G.SRT.A.3 $\square$ G.SRT.B. 4 • Prove theorems involving similarity <br> G.CO.C. 9 $\square$ G.SRT.B.5  | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 2 Reason abstractly and quantitatively. |
| Unit 3: <br> Suggested <br> Educational <br> Resources | G.SRT.A. 1 Dilating a Line  <br> G.SRT.A. 2 Are They Similar? G.CO.C.10 Midpoints of Triangle Sides <br> G.SRT.A. 2 Similar Triangles <br> G.CO.C.10 Sum of angles in a triangle  <br> G.SRT.A. 3 Similar Triangles G.CO.C.11 Midpoints of the Sides of a Parallelogram <br> G.CO.C. 9 Congruent Angles made by parallel G.CO.C.11 Is this a parallelogram? <br> lines and a transverse G.SRT.B.4 Joining two midpoints of sides of a triangle <br> G.CO.C. 9 Points equidistant from two points in G.SRT.B.4 Pythagorean Theorem <br> the plane  <br>  $\quad$.  | MP. 3 Construct viable arguments \& critique the reasoning of others. <br> MP. 4 Model with mathematics. <br> MP. 5 Use appropriate tools strategically. |
| Unit 4 <br> Trig Ratios \& Geometric Equations |  | MP. 6 Attend to precision. <br> MP. 7 Look for and make use of structure. <br> MP. 8 Look for and express regularity in repeated reasoning. |
| Unit 4: <br> Suggested <br> Educational <br> Resources | G.GPE.B.4,5 A Midpoint Miracle <br> G.GPE.B. 5 Slope Criterion for Perpendicular <br> G.GPE.B. 7 Triangle Perimeters <br> G.SRT.C. 6 Defining Trigonometric Ratio <br> G.SRT.C. 7 Sine and Cosine of Complimentary Angles |  |

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| Unit 1 Algebra Lab |  |  |  |
| :---: | :---: | :---: | :---: |
| Content \& Practice Standards | Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context. <br> A.SSE.A.1a. Interpret parts of an expression, such as terms, factors, and coefficients. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - identify different parts of an expression, including terms, factors and constants. <br> - explain the meaning of parts of an expression in context. <br> Learning Goal 3: Interpret terms, factors, coefficients, and other parts of expressions in terms of a context. | Give an example of two like terms and two unlike terms. Explain why they would or would not be classified as like terms. |
| A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | MP 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Equations and inequalities describe relationships. <br> - Equations can represent real-world and mathematical problems. <br> Students are able to: <br> - identify and describe relationships between quantities in word problems. <br> - create linear equations in one variable. <br> - create linear inequalities in one variable. <br> - use equations and inequalities to solve real world problems. <br> - explain each step in the solution process. <br> Learning Goal 4: Create linear equations and inequalities in one variable and use them in contextual situations to solve problems. Justify each step in the process and the solution. | Jennifer had \$30 to spend on herself. She spent $1 / 5$ of the money on a sandwich, $1 / 6$ for a ticket to a museum, and $1 / 2$ of it on a book. How much money does Jennifer have left over? <br> Tim is choosing between two cell phone plans that offer the same amount of free minutes. Sprint's plan charges $\$ 39.99$ per month with additional minutes costing $\$ 0.45$. Verizon's plan costs $\$ 44.99$ with additional minutes at $\$ 0.40$. How many additional minutes, a, will it take for the two plans to cost the same? |

- A.REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $\mathrm{V}=\mathrm{IR}$ to highlight resistance R .

|  |  |
| :--- | :--- |
| $\square$ | $\begin{array}{l}\text { S.ID.B.6. Represent data } \\ \text { on two quantitative }\end{array}$ | variables on a scatter plot, and describe how the variables are related.

S.ID.B.6a. Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or

MP 2 Reason abstractly and quantitatively.

MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.

MP. 1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively. MP. 4 Model with mathematics.

MP. 5 Use appropriate tools strategically.

Concept(s).

- Literal equations can be rearranged using the properties of equality.
Students are able to.
- solve linear equations with coefficients represented by letters in one variable.
- use the properties of equality to justify steps in solving linear equations.
- solve linear inequalities in one variable.
- rearrange linear formulas and literal equations, isolating a specific variable.

Learning Goal 2. Solve linear equations and inequalities in one variable (including literal equations); justify each step in the process.

## Concept(s):

- Scatter plots represent the relationship between two variables.
- Scatter plots can be used to determine the nature of the association between the variables.
- Linear models may be developed by fitting a linear function to approximately linear data.
- The correlation coefficient represents the strength of a linear association.
Students are able to:
- distinguish linear models representing

Rewrite the following formula to highlight the variable " $h$ "

$$
A=\frac{\left(b_{1}+b_{2}\right) \cdot h}{2}
$$

Solve the following inequality for $y$, where $a, b$, and $c$ are positive real numbers. Show all work and justify each step in the work with mathematical reasoning.

$$
a x-b y>c
$$

Solve the following equation. Show all work and justify each step in the work with a mathematical reason.

$$
\frac{1}{3}(2 x-5)-2=\frac{1}{2}(x-2)
$$

Solve the following inequality
$3(x-4) \leq 8 x+13$ for $x$
6. Make a scatier plot of the data, and draw a line of best fit. Then use the data to
predict the percentage of American homeowners in 1955 ,

Preaiction


The graph shows the altitude of an
choose a function suggested by the context. Emphasize linear, quadratic, and exponential models

## S.ID.B.6c. Fit a linear

 function for a scatter plot that suggests a linear association.S.ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.C.9. Distinguish between correlation and causation.
A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y$ $=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations Include cases where $\mathrm{f}(\mathrm{x})$ and/or $g(x)$ are linear, polynomial, rational, absolute value
exponential, and logarithmic functions.*

MP. 6 Attend to precision.

MP. 1 Make sense of problems and persevere in solving them.

MP. 3 Construct viable arguments and critique the reasoning of others

MP. 5 Use appropriate tools strategically.
approximately linear data from linear equations representing "perfectly" linear relationships.

- create a scatter plot and sketch a line of best fit
- fit a linear function to data using technology.
- solve problems using prediction equations.
- interpret the slope and the intercepts of the linear model in context.
- determine the correlation coefficient for the linear model using technology.
- determine the direction and strength of the linear association between two variables

Learning Goal 6: Represent data on a scatter plot, describe how the variables are related and use technology to fit a function to data.
Learning Goal 7: Interpret the slope, intercept, and correlation coefficient of a data set of a linear model; distinguish between correlation and causation.

## Concept(s):

- $y=f(x), y=g(x)$ represent a system of equations.
- Systems of equations can be solved graphically (8.EE.C.8).

Students are able to:

- explain the relationship between the x-coordinate of a point of intersection and the solution to the equation $f(x)=g(x)$ for linear equations $y=f(x)$ and $\mathrm{y}=\mathrm{g}(\mathrm{x})$.
- find approximate solutions to the system by making a table of values, graphing, and finding successive approximations.

Learning Goal 8: Explain why the solutions of the equation $f(x)=g(x)$ are the $x$-coordinates of the
airplane as it comes in for a landing. In comparing the time to the altitude, find the rate of change and distinguish the correlation.

| Time(seconds) | Altitude (feet) |
| :--- | :--- |
| 0 | 30,000 |
| 5 | 28,192 |
| 10 | 25,962 |
| 15 | 24,450 |
| 20 | 21,320 |
| 25 | 18,780 |
| 30 | 15,256 |

For the functions defined below, fill in the table of values and circle the row of the table that indicates the solution to $\mathrm{f}(\mathrm{x})=$ $g(x)$. Then give the solution to $f(x)=g(x)$.
$f(x)=3 x-8$,
$g(x)=0.5 x+7$

| $x$ | $\mathrm{f}(\mathrm{x})$ | $\mathrm{g}(\mathrm{x})$ |
| :---: | :---: | :---: |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |

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Variable, algebraic expression, equation, evaluate, simplify, exponent, base, power, rational number, irrational number, real numbers, inequality, opposites, absolute value, coordinate plane, coordinates, ordered pair, function, function rule, domain, range, dependent variable, independent variable, scatter plot, correlation, line of best fit, measures of central tendency, additive inverse, matrix, multiplicative inverse, reciprocal, term, coefficient, equivalent equations, solution, consecutive integers, equivalent inequalities, relation, vertical-line test, function notation, continuous data, discrete data, direct variation, inverse variation, inductive reasoning, conjecture, rate of change, slope, linear function, linear equation, slopeintercept form, standard form, y-intercept, x-intercept, point-slope form, parallel and perpendicular lines.

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 1 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos www.desmos.com
d. Scavenger Hunt
e. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
a. Use of Higher Level Questioning Techniques
b. Extension/Challenge Questions
c. Provide Assessments at a Higher Level of Thinking
d. Desmos Linear Activities - https://teacher.desmos.com/linear

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

## - Use of APEX

## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them..

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CRP11. Use technology to enhance productivity.

### 9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
Career \& Technical Education Content Area: $21^{\text {st }}$ Century Life and Careers Standards
9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

Interdisciplinary Connections/Global Perspective: Research the different units of measure used around the world. Write an equation to convert each of the different units of measure used for length into the customary system used in the United States. Describe some of the benefits or drawbacks of the different systems of measurement.

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Unit 2

| Unit 2 Algebra Lab |  |  |  |
| :---: | :---: | :---: | :---: |
| Content \& Practice Standards | Standards for <br> Mathematical Practice | Critical Knowledge \& Skills | Examples |
| A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 4 Model with mathematics. | Concept(s): <br> - Systems of equations can be solved exactly (algebraically) and approximately (graphically). <br> Students are able to: <br> - identify and define variables representing essential features for the model. <br> - model real world situations by creating a system of linear equations. <br> - solve systems of linear equations using the elimination or substitution method. <br> - solve systems of linear equations by graphing. <br> - interpret the solution(s) in context. <br> Learning Goal 1: Solve multistep contextual problems by identifying variables, writing equations, and solving systems of linear equations in two variables algebraically and graphically. | A garden supply store sells two types of lawn mowers. Total sales of mowers for the year were $\$ 8,379.70$. the total number of mowers sold was 30 . The small mowers cost $\$ 249.99$ and the large mowers cost $\$ 329.99$. <br> a. Write two equations clearly defining the variables to represent the above. <br> b. Find the number of each type of lawn mower sold. <br> A.REI.C. 6 Cash Box <br> Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said, <br> "I wonder whether the dollar belongs inside the cash box or not." <br> The price of tickets for the dance was 1 ticket for $\$ 5$ (for individuals) or 2 tickets for $\$ 8$ (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. <br> Does the dollar belong inside the cash box or not? |
| A.REI.D.12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of | MP. 1 Make sense of problems and persevere in solving them. <br> MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - model real world situations by creating a system of linear inequalities given a context. <br> - interpret the solution(s) in context. | A clothing manufacturer has 1, 000 yd. of cotton to make shirts and pajamas. A shirt requires 1 yd . of fabric, and a pair of pajamas requires 2 yd . of fabric. It takes 2 hr . to make a shirt and 3 hr . to make the pajamas, and there are $1,600 \mathrm{hr}$. available to make the clothing. <br> a. What are the variables? <br> Number of shirts made and number of |

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the corresponding half-planes. mathematics.

- A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. strategically.

MP. 5 Use appropriate tools

MP. 6 Attend to precision.

MP 2 Reason abstractly and quantitatively.

MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.

Learning Goal 2: Graph linear inequalities and systems of linear inequalities in two variables and explain that the solution to the system of inequalities is the intersection of the corresponding half-planes.

## pajamas made.

## b. What are the constraints?

How much time the manufacturer has and how much material is available.

## c. Write inequalities for the constraints.

Let $x=$ number of shirts, and let $y=$
number of pajamas.
$x \geq 0$ and $y \geq 0$
$x+2 y \leq 1000$
$2 x+3 y \leq 1600$
d. Graph the inequalities and shade the solution set.


## e. What does the shaded region represent?

The various combinations of shirts and pajamas that it would be possible for the manufacturer to make.
f. The shaded region in a problem of this type is sometimes called the feasible region. Why does this name make sense?

This is the region that represents the number of shirts and pajamas that he can feasibly make given the constraints.

Jerome is constructing a table of values that satisfies the definition of a function.
the definition of a function? Select all that apply.
$\begin{array}{llllll}\text { A. }-5 & \text { B. }-1 & \text { C. } 0 & \text { D. } 2 & \text { E. } 11 & \text { F. } 17\end{array}$
the equation $\mathrm{y}=\mathrm{f}(\mathrm{x})$.
F.IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.A.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For
example, the Fibonacci sequence is defined recursively by $f(0)=f(1)=1$, $f(n+1)=f(n)+f(n-1)$ for $n \geq$ 1.

- evaluate functions for given inputs in the domain.
- explain statements involving function notation in the context of the problem.

Learning Goal 3: Explain the definition of a function, including the relationship between the domain and range. Use function notation, evaluate functions and interpret statements in context.
MP 2 Reason abstractly and $\quad$ Concept(s): quantitatively.

MP 4. Model with mathematics

MP. 1 Make sense of problems and persevere in solving them.

MP. 5 Use appropriate tools strategically.

MP. 6 Attend to precision.
MP. 7 Look for and make use of structure.

- Sequences are functions, sometimes defined and represented recursively.
- Sequences are functions whose domain is a subset of integers.
Students are able to:
- create arithmetic and geometric sequences from verbal descriptions.
- create arithmetic sequences from linear functions.
- create geometric sequences from exponential functions.
- identify recursively defined sequences as functions.
- create linear and exponential functions given
- a graph;
- a description of a relationship;
- a table of values.

Learning Goal 5: Write linear and exponential functions given a graph, table of values, or written description; construct arithmetic and geometric sequences.

MP 2 Reason abstractly and quantitatively.
A.SSE.A.1. Interpret expressions that represent a quantity in terms of its

MP. 4 Model with

## Concept(s): No new concept(s) introduced

Students are able to:

- given a context, write an explicit expressions, a

Write a geometric sequence. You must include the first four terms of your sequence. Identify your common ratio then write an equation to represent the rule.

In (a)-(e), say whether the quantity is changing in a linear or exponential fashion. Write the equation of the function.
a. A savings account, which earns no interest, receives a deposit of $\$ 723$ per month.
b. The value of a machine depreciates by $17 \%$ per year.
c. Every week, $9 / 10$ of a radioactive substance remains from the beginning of the week.
d. A liter of water evaporates from a swimming pool every day.
e. Every 124 minutes, $1 / 2$ of a drug dosage remains in the body.

All exponential functions are in the form $\mathrm{y}=$ $\mathrm{a}(\mathrm{b})^{\mathrm{x}}$.

What values of $b$ make it an exponential

| context <br> A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients. <br> A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of $P$ and a factor not depending on $P$. <br> *[Algebra 1 limitation: exponential expressions with integer exponents] | mathematics | recursive process or steps for calculation for linear and exponential relationships. <br> - interpret parts of linear and exponential functions in context. <br> Learning Goal 6: Write explicit expressions, recursive processes and steps for calculation from a context that describes a linear or exponential relationship between two quantities. | growth function? <br> What values of $b$ make it an exponential decay function? <br> If you have $\$ 200$ to invest for 10 years, would you rather invest your money in a bank that pays $7 \%$ simple interest or in a bank that pays $5 \%$ interest compounded annually? Is there anything you could change in the problem that would make you change your answer? <br> Show how you arrived at your answer using a recursive formula and then using an explicit formula. |
| :---: | :---: | :---: | :---: |
| $\square$ F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *[Focus on exponential functions] F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive | MP 2 Reason abstractly and quantitatively. <br> MP. 4 Model with mathematics. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - given a verbal description of a relationship, sketch linear and exponential functions. <br> - identify intercepts and intervals where the function is positive/negative. <br> - interpret parameters in context. <br> - determine the practical domain of a function. <br> Learning Goal 8: Sketch graphs of linear and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context. | Examine the graphs of $f(x)=3^{x}$ and $g(x)=5 x$, shown below. <br> a. Estimate the values of $x$ for which $f(x)$ is greater than $g(x)$. <br> b. Estimate the values of $x$ for which $g(x)$ is greater than $f(x)$. |

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| integers would be an appropriate domain for the function |  |  |  |
| :---: | :---: | :---: | :---: |
| $\square$ F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 8 Look for and express regularity in repeated reasoning. | Concept(s): <br> - Rate of change of non-linear functions varies. <br> Students are able to: <br> - compare key features of two linear functions represented in different ways. <br> - compare key features of two exponential functions represented in different ways. <br> - calculate the rate of change from a table of values or from a function presented symbolically. <br> - estimate the rate of change from a graph. <br> Learning Goal 9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <br> Learning Goal 10: Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph. | In 2007, Zack bought a new car for $\$ 17,500$. The table below shows the value of the car between 2007 and 2012. <br> Part A. Calculate the average rate of change of the value of the car between 2007 and 2008. Explain what your answer means in terms of the car's value over this interval. <br> Part B. Calculate the average rate of change of the value of the car between 2008 and 2012. Explain what your answer means in terms of the car's value over this interval. <br> Part C. Compare the values from Part A and Part B. What can you conclude based on this comparison along with the data in the table in terms of the car's value over the time period shown in the table? Use words, numbers and/or pictures to show your work. |
| F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 5 Use appropriate tools | Concept(s): <br> - Piecewise-defined functions may contain discontinuities. <br> - Absolute value functions are piecewise functions. | Graph $\mathrm{f}(\mathrm{x})=\|x\|$ and the resulting graphs of the expanded functions. Compare and contrast the behavior of these graphs to $\mathrm{f}(\mathrm{x})=$ $x$ and its expanded versions $f(x)=a x+c$. |

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| for more complicated cases. <br> F.IF.C.7b. Graph piecewise-defined functions, including step functions and absolute value functions. | strategically. <br> MP. 6 Attend to precision. | Students are able to: <br> - graph linear, piecewise-defined functions. <br> - graph more complicated cases of functions using technology. <br> - identify and describe key features of the graphs of piecewise-defined functions . <br> Learning Goal 11: Graph linear and piecewise-defined functions (including step and absolute value functions) expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph. | At the beginning of the week, Jessie had $\$ 500$ in her bank account. She deposited a check for $\$ 50$ on Tuesday and then paid $\$ 250$ in rent on Wednesday. On Friday, Jessie deposited $\$ 200$ in the account and then on Saturday she paid $\$ 50$ for groceries from her bank account. Jessie made the following graph for the balance in her bank account during this week: <br> a. Is the depiction of how the account balance varies over the week accurate? Explain. <br> b. How can Jessie graphically represent the bank account balance in a way that better shows how it changes? |
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## Unit 2 Vocabulary

System, substitution, elimination, linear combination, consistent and inconsistent system, dependent and independent system, infinitely many solutions, no solution, half-plane, exponent, negative exponent power, base, order of magnitude, Power of a Product/Quotient Property, Product/Quotient of a Power Property, Power of a Power Property, reciprocal, scientific notation, exponential function, exponential growth, exponential decay, compound interest, initial amount, growth/decay factor, growth/decay rate, time.

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 2 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos www.desmos.com
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
e. Use of Higher Level Questioning Techniques
f. Extension/Challenge Questions
g. Provide Assessments at a Higher Level of Thinking
h. Desmos Linear Activities - https://teacher.desmos.com/linear
i. Desmos Exponential Activities - https://teacher.desmos.com/exponential

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- Use of APEX


## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation
9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.

## Career \& Technical Education Content Area: 21 ${ }^{\text {St }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.

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9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.

## Interdisciplinary Connections:

- Science: Use projectile motion equations to model the height of objects when affected by gravity.
- Social Studies: Write functions to determine rate of return on stock investments.

Media Literacy: Look through the local newspaper or magazine and find 3 advertisements for local or national banks. Compare the advertised rates for checking or savings accounts. Use an exponential equation to compare the different offers and determine which will lead to the greatest interest after 1,5 , and 10 years.

## Unit 3

| Unit 3 Geometry |  |  |  |
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| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| G.SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor: <br> G.SRT.A.1a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. <br> G.SRT.A.1b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | MP. 1 Make sense of problems and persevere in solving them MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. <br> MP. 8 Look for and express regularity in repeated reasoning. | Concept(s): <br> - Dilation of a line that passes through the center of dilation results in the same line. <br> - Dilation of a line that does not pass through the center of dilation results in a line that is parallel to the original line. <br> - Dilation of a line segment results in a longer line segment when, for scale factor $k,\|k\|$ is greater than 1. <br> - Dilation of a line segment results in a shorter line segment when, for scale factor $\mathrm{k},\|\mathrm{k}\|$ is less than 1 . <br> Students are able to: <br> - perform dilations in order to verify the impact of dilations on lines and line segments. <br> Learning Goal 1: Verify the properties of dilations given by a center and a scale factor. | Draw the image of the following triangle after a dilation centered at the origin with a scale |


| $\square$ G.SRT.A.2. Given two figures, use the definition of similarity to decide if they are similar; explain using the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. <br> G.SRT.A.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 8 Look for and express regularity in repeated reasoning. | Concept(s): <br> - Similarity transformations are used to determine the similarity of two figures. <br> Students are able to: <br> - given two figures, determine if they are similar. <br> - explain the meaning of similarity for triangles. <br> - explain AA criterion and its relationship to similarity transformations and properties of triangles <br> Learning Goal 2: Use the definition of similarity in terms of similarity transformations to decide if two given figures are similar and explain, using similarity transformations, the meaning of triangle similarity. | (a) Which sequence(s) of transformations will map Figure $A$ onto Figure exactly? Choose all that apply, <br> Rotate Figure A clockwise $180^{\circ}$ about the origin, and then dilate that result with scale factor 2 centered at the origin. <br> $\square$ Dilate Figure A with scale factor 2 centered at the origin, and then rotate that result clockwise $90^{\circ}$ about the origin. <br> Dilate Figure A with scale factor 2 centered at the origin, and then translate that result up 10 units. <br> $\square$ Dilate Figure A with scale factor 2 centered at the origin, and then reflect that result over the $x$-axis. <br> $\square$ None of these <br> (b) Are Figure A and Figure B similar? <br> $\bigcirc$ Yes No |
| :---: | :---: | :---: | :---: |
| Unit 3 Geometry |  |  |  |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| G.CO.C.9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. <br> G.CO.C.10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to $180^{\circ}$; base angles of isosceles triangles are congruent; the segment | MP. 3 Construct viable arguments and critique the reasoning of others. <br> MP. 6 Attend to precision. | Concept(s): <br> - A formal proof may be represented with a paragraph proof or a two-column proof. <br> Students are able to: <br> - construct and explain proofs of theorems about lines and angles including: <br> - vertical angles are congruent; <br> - angle and segment addition postulate <br> - complementary and supplementary angles <br> - algebraic and midpoint proofs <br> - congruence of alternate interior angles; <br> - congruence of corresponding angles; <br> - and points on a perpendicular bisector of a line segment are exactly those equidistant from the | $\begin{aligned} \frac{1}{2} x-5 & =10 & & \text { Given } \\ 2\left(\frac{1}{2} x-5\right) & =20 & & \text { a. } \frac{?}{?} \\ x-10 & =20 & & \text { b. } \frac{?}{?} \\ x & =30 & & \text { c. } ? \end{aligned}$ |

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| joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. <br> G.CO.C.11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. |  | segment's endpoints. <br> - construct and explain proofs of theorems about triangles including: <br> - sum of interior angles of a triangle; <br> - congruence of base angles of an isosceles triangle; <br> - the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; <br> - and the medians of a triangle meet at a point. <br> - construct and explain proofs of theorems about parallelograms including: <br> - opposite sides are congruent; opposite angles are congruent; <br> - the diagonals of a parallelogram bisect each other; <br> - and rectangles are parallelograms with congruent diagonals <br> Learning Goal 4: Construct and explain formal proofs of theorems involving lines, angles, triangles, and parallelograms. | $\angle C D E$ and $\angle E D F$ are supplementary $\begin{aligned} m \angle C D E+m \angle E D F & =180 \\ x+(3 x+20) & =180 \\ 4 x+20 & =180 \\ 4 x & =160 \\ x & =40 \end{aligned}$ | \&s that form a linear pair are supplementary <br> a. ? <br> b. ? <br> c. ? <br> d. ? <br> e. ? |
| :---: | :---: | :---: | :---: | :---: |
| Unit 3 Geometry |  |  |  |  |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examp |  |

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| $\square$ G.SRT.B.4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity | MP. 2 Reason abstractly and quantitatively. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - construct and explain proofs of theorems about triangles including: <br> - a line parallel to one side of a triangle divides the other two sides proportionally; <br> - and the Pythagorean Theorem (using triangle similarity). <br> Learning Goal 5: Prove theorems about triangles. | In $\triangle Q R S, \overline{Q R} \\| \overline{T U}$. Given that $S Q=27, S T=12$, and $T U=20$, find $Q R$. |
| :---: | :---: | :---: | :---: |
| $\square$ G.SRT.B.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | MP. 7 Look for and make use of structure. | Concept(s): <br> - Corresponding parts of congruent triangles are congruent (СРCTC). <br> Students are able to: <br> - prove geometric relationships in figures using criteria for triangle congruence. <br> - solve problems using triangle congruence criteria (SSS, ASA, SAS, HL). <br> - solve problems using triangle similarity criteria (AA). <br> Learning Goal 6: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | Use the given information to prove that $\overline{V U} \cong \overline{S R}$. |

## Vocabulary

Property of equality, addition property, subtraction property, multiplication property, division property, substitution property, symmetric property, reflexive property, transitive property, distributive property, property of congruence, two column proof, corresponding angles and their converse, alternate interior angles and their converse, alternate exterior angles and their converse, same side interior angles and their converse, triangle angle sum theorem, exterior angle, remote interior angles, SSS, SAS, ASA, AAS, HL, isosceles triangle, equilateral triangle, equiangular triangle, CPCTC, triangle mid-segment, perpendicular bisector, median, centroid, altitude, parallelogram.

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 3 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos https://teacher.desmos.com/transformations
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.
h. Repetition and practice
i. Pair Visual Prompts with Verbal Presentations
j. Provide Formulas
k. Check Use of Agenda
4. Gifted and Talented Students.
j. Use of Higher Level Questioning Techniques
k. Extension/Challenge Questions
5. Provide Assessments at a Higher Level of Thinking
m. Exploration Problems/Proofs

## New Jersey Student Learning Standards - Technology

8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- The students use the Geometer's Sketchpad to draw and measure angles that model the relationships they learn in this unit. The
students use their drawings from the Geometer's Sketchpad to create a PowerPoint presentation reviewing angle classifications and relationships.


## Career Readiness Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: Career Preparation
9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.
9.2.8.B.4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.
9.2.8.B. 5 Analyze labor market trends using state and federal labor market information and other resources available online.
9.2.8.B. 7 Evaluate the impact of online activities and social media on employer decisions.

## Career \& Technical Education Content Area: $\mathbf{2 1}^{\text {st }}$ Century Life and Careers Standards

9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data.
9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society.
9.3.ST-ET. 2 -- Display and communicate STEM information.
9.3.ST-ET. 4 -- Apply the elements of the design process

Interdisciplinary Connections: The students use a paragraph proof instead of a two-column proof as a writing exercise.

Media Literacy: Students will examine the use of Geometric figures in advertising. Questions to be considered include -Is there significance in the figure chosen, either symbolically or figuratively? Was the selection of a particular figure intended to influence the consumer? In what ways might it influence the consumer?

## Unit 4



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| Unit 4 Geometry |  |  |  |
| :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| G.GPE.B.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.B.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 2 Reason abstractly and quantitatively. MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - locate the point on a directed line segment that creates two segments of a given ratio. <br> - find perimeters of polygons using coordinates, the Pythagorean theorem and the distance formula. <br> - find areas of triangle and rectangles using coordinates. <br> Learning Goal 3: Find the point on a directed line segment between two given points that partitions the segment in a given ratio and use coordinates to compute perimeters of polygons and areas of triangles and rectangles. | The diagram shows $\overline{M N}$ graphed on a coordinate plane. <br> Point $P$ lies on $\overline{M N}$ and is $\frac{3}{4}$ of the way from $M$ to $N$. What are the coordinates of point $P$ ? <br> Enter your answer in the space provided. Enter only your answer. $(\square, \square)$ <br> Point $A$ is located at -3 , and point $B$ is located at 19. <br> Select a point on the number line between $A$ and $B$ such that the distance from $A$ to the point is $\frac{3}{11}$ of the distance from $A$ to $B$ <br> Select a place on the number line to plot the point. |

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| Unit 4 Geometry |  |  |  |
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| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| G.SRT.C.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | MP. 7 Look for and make use of structure. | Concept(s): <br> - Side ratios in right triangles are properties of the angles in the triangle. <br> Students are able to: <br> - show and explain that definitions for trigonometric ratios derive from similarity of right triangles. <br> Learning Goal 4: Show and explain that definitions for trigonometric ratios derive from similarity of right triangles. | Solve for $x$ in the triangle. Round your answer to the nearest tenth. |

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| Unit 4 Geometry |  |  |  |
| :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| $\square$ G.SRT.C.7. Explain and use the relationship between the sine and cosine of complementary angles G.SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 2 Reason abstractly and quantitatively. MP. 5 Use appropriate tools strategically. <br> MP. 6 Attend to precision. <br> MP. 7 Look for and make use of structure. | Concept(s): <br> - Relationship between sine and cosine of complementary angles <br> Students are able to: <br> - determine and compare sine and cosine ratios of complementary angles in a right triangle. <br> - solve right triangles (determine all angle measures and all side lengths) using trigonometric ratios and the Pythagorean Theorem. <br> Learning Goal 5: Explain and use the relationship between the sine and cosine of complementary angles; use trigonometric ratios and the Pythagorean Theorem to compute all angle measures and side lengths of triangles in | The figure below is a right triangle with side lengths $x, y$, and $z$. <br> Suppose that $m \angle X$ does not equal $m \angle Y$. <br> Complete the following. <br> Part 1: Use $x, y$, and $z$ to fill in the blanks. <br> Make sure to use the appropriate upper-case or lower-case letters. $\begin{array}{ll} \sin X=\frac{\square}{\square} & \sin Y=\frac{\square}{\square} \\ \cos X=\frac{\square}{\square} & \cos Y=\frac{\square}{\square} \end{array}$ <br> Part 2: In $\triangle X Y Z, \angle X$ and $\angle Y$ are (Choose one) - <br> Part 3: Select all of the true statements. <br> $\sin X=\sin Y$ <br> $\sin X=\cos Y$ <br> $\cos X=\cos Y$ <br> $\cos X=\sin Y$ <br> None of the above is true. <br> Part 4: Fill in the blank. $\sin \left(58^{\circ}\right)=\cos \left(\square^{\circ}\right)$ |

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| Unit 4 Geometry |  |  |  |
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| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
|  |  | applied problems. |  |
| (©) G.GPE.A.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | MP. 6 Attend to precision. <br> MP. 7 Look for and make use of structure. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - given the center and radius, derive the equation of a circle (using the Pythagorean Theorem). <br> - given an equation of a circle in any form, use the method of completing the square to determine the center and radius of the circle. <br> Learning Goal 6: Derive the equation of a circle of given the center and radius using the Pythagorean Theorem. Given an equation, complete the square to find the center and radius of the circle. | The circle below has center $P$. <br> The point $(x, y)$ is on the circle as shown. <br> (a) Find the following. <br> Radius: $\square$ units <br> Center: $\square$ , ]) <br> Value of $a$ : (Choose one) <br> Value of $b$ : (Choose one) <br> (b) Use the Pythagorean Theorem to write an equation relating the side lengths of the right triangle. Write your answer in terms of $x$ and $y$ (with no other letters). $(\square)^{2}+(\square)^{2}=(\square)^{2}$ |

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| Unit 4 Geometry |  |  |  |
| :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| (0) G.C.A.1. Prove that all circles are similar. | MP. 3 <br> Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically. | Concept(s): <br> - Similarity of all circles Students are able to: <br> - construct a formal proof of the similarity of all circles. <br> Learning Goal 7: Prove that all circles are similar | In the figure below, the solid circle has center $(7,4)$ and radius 3 . The dashed circle has center $(3,8)$ and radius 2 . <br> Use the transformation tools given to move the solid circle exactly onto the dashed circle. Then answer the parts below. <br> (a) Fill in the blanks to describe the transformations necessary to move the solid circle exactly onto the dashed circle. <br> Translate the solid circle Choose one • by $\square$ unit(s) and Choose one $~$ by $\square$ unit(s). <br> Dilate the solid circle by a scale factor of $\square$. <br> (b) Are the original solid circle and the dashed circle similar? <br> - No |

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| Unit 4 Geometry |  |  |  |
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| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| (0) G.C.A.2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. | MP. 1 Make sense of problems and persevere in solving them. <br> MP. 5 Use appropriate tools strategically. | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - use the relationship between inscribed angles, radii and chords to solve problems. <br> - use the relationship between central, inscribed, and circumscribed angles to solve problems. <br> - identify inscribed angles on a diameter as right angles. <br> - identify the radius of a circle as perpendicular to the tangent where the radius intersects the circle. <br> - find arc length and area of a sector <br> Learning Goal 8: Identify and describe relationships among inscribed angles, radii, and chords; use these relationships to solve problems. | In the circle below, $\overline{Q S}$ is a diameter. Suppose $m \overparen{Q R}=62^{\circ}$ and $m \angle Q R T=66^{\circ}$. Find the following. <br> (a) $m \angle R Q S$ <br> (b) $m \angle S R T$ <br> In the figure below, the segments $\overline{D E}$ and $\overline{D F}$ are tangent to the circle centered at $O$. Given that $O E=3.6$ and $O D=6$, find $D F$. <br> The circle below has center $O$, and its radius is 7 ft . Given that $m \angle A O B=100^{\circ}$, find the length of the arc $\overparen{A D B}$ and the area of the shaded region. <br> Give exact answers in terms of $\pi$, and be sure to include the correct units in your answer. |

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| Unit 4 Geometry |  |  |  |
| :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| G.C.B.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | MP. 2 Reason abstractly and quantitatively. MP. 3 <br> Construct viable arguments and critique he reasoning of others. | Concept(s): <br> - A proportional relationship exists between the length of an arc that is intercepted by an angle and the radius of the circle. <br> Students are able to: <br> - use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius. <br> - define radian measure of an angle as the constant of proportionality when the length of the arc intercepted by an angle is proportional to the radius. <br> - derive the formula for the area of a sector. <br> - compute arc lengths and areas of sectors of circles. <br> Learning Goal 7: Find arc lengths and areas of sectors of circles; use similarity to show that the length of the arc | The circle below with center $C$ is divided into 8 equal slices. <br> Central angle $\angle A C B$ intercepts $\overparen{A B}$, forming the shaded sector. <br> Complete the statements below. <br> Give your answers as exact values, not decimal approximations. <br> Complete the statements below. <br> Give your answers as exact values, not decimal approximations. <br> (a) From the figure we see that $m \angle A C B=45^{\circ}$. <br> The measure of $\angle A C B$ is (Choose one) * the measure of $\overparen{A B}$. <br> So, $m \overparen{A B}=\square^{\circ}$. <br> This gives the following equation. $\frac{m \overparen{A B}}{360^{\circ}}=\square$ <br> (b) Find the ratio of the area of the sector to the area of the circle. $\frac{\text { Area of sector }}{\text { Area of circle }}=\square$ <br> (c) Choose the equation that is true. $\begin{array}{ll} \frac{\text { Area of sector }}{\text { Area of circle }}=\frac{360^{\circ}}{m \overparen{A B}} \quad \frac{\text { Area of sector }}{\text { Area of circle }}=8 \cdot \frac{m \overparen{A B}}{360^{\circ}} \\ \frac{\text { Area of sector }}{\text { Area of circle }}=\frac{m \overparen{A B}}{360^{\circ}} \quad \frac{\text { Area of sector }}{\text { Area of circle }}=m \overparen{A B} \end{array}$ <br> (d) Using $\pi r^{2}$ for the area of the circle, choose the equation that gives a formula for the area of the sector. $\begin{aligned} & \text { (- Area of sector }=\frac{m \overparen{A B}}{360^{\circ}} \cdot 8 \pi r^{2} \\ & \text { O Area of sector }=\frac{m \overparen{A B}}{360^{\circ}} \cdot \pi r^{2} \\ & \text { Area of sector }=m \overparen{A B} \cdot \pi r^{2} \end{aligned} \text { O Area of sector }=\frac{360^{\circ}}{m \overparen{A B}} \cdot \pi r^{2} .$ |

Algebra Lab

| Unit 4 Geometry |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills |  | Examples |
|  |  | intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector. |  |  |
| (0) G.C.A.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | MP. 3 <br> Construct viable arguments and critique the reasoning of others. <br> MP. 5 Use appropriate tools strategically | Concept(s): No new concept(s) introduced <br> Students are able to: <br> - construct the inscribed circle of a triangle. <br> - construct the circumscribed circle of a triangle. <br> - prove properties of the angles of a quadrilateral that is inscribed in a circle. <br> Learning Goal 9: Prove the properties of angles for a quadrilateral inscribed in a circle and construct inscribed and circumscribed circles of a triangle using geometric tools and geometric software. | Quadrilateral $S T U V$ is inscribed in | cle $R$. Complete the following. <br> (a) Choose the correct expression to finish each statement. $\begin{aligned} & m \angle T=\text { (Choose one) } \mathbf{V} \\ & m \angle V=\text { (Choose one) } \mathbf{v} \end{aligned}$ <br> (b) Choose the correct expressions for the sum of $m \angle T$ and $m \angle V$. $\begin{aligned} m \angle T+m \angle V & =\text { (Choose one) } \\ & =\text { (Choose one) } \end{aligned}$ <br> (c) Fill in the blank with the correct number. $m \angle T+m \angle V=\square^{\circ}$ <br> (d) From the steps above, classify $\angle T$ and $\angle V$. $\angle T$ and $\angle V$ are supplementary. $\angle T$ and $\angle V$ are neither supplementary nor complementary. $\angle T$ and $\angle V$ are complementary. |

Algebra Lab

| Unit 4 Geometry |  |  |  |
| :---: | :---: | :---: | :---: |
| Content Standards | Suggested Standards for Mathematical Practice | Critical Knowledge \& Skills | Examples |
| Vocabulary |  |  |  |
| Ratio, proportion, cross products property, similar figures, similar polygons, scale factor, scale drawing, geometric mean, Pythagorean triple, trigonometric ratios, sine, cosine, tangent, angle of elevation, angle of depression, circle, center, diameter, radius, congruent circles, central angle, semicircle, minor arc, major arc, adjacent arcs, circumference, pi, arc length congruent arcs. |  |  |  |

## Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
a. See Unit 4 Suggested Open Educational Resources
b. Two quizzes/Two Tests
c. Desmos https://teacher.desmos.com
d. Open Middle Problems http://www.openmiddle.com/
2. English Language Learners.
a. Read written instructions
b. Students may be provided with note organizers/study guides to reinforce key topics.
c. Model and provide examples
d. Extended time on assessments when needed.
e. Establish a non-verbal cue to redirect student when not on task.
f. Students may use a bilingual dictionary.
g. Pair Visual Prompts with Verbal Presentations
h. Highlight Key Words \& Formulas
3. Special Education/504 Students.
a. Students may be provided with note organizers / study guides to reinforce key topics.
b. Extended time on assessments when needed.
c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.

| h. Repetition and practice <br> i. Pair Visual Prompts with Verbal Presentations <br> j. Provide Formulas <br> k. Check Use of Agenda <br> 4. Gifted and Talented Students. <br> n. Use of Higher Level Questioning Techniques <br> o. Extension/Challenge Questions <br> p. Provide Assessments at a Higher Level of Thinking <br> q. Exploration Problems/Proofs |
| :---: |
| New Jersey Student Learning Standards - Technology <br> 8.1.8.A.5 - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems. <br> - The students use the Geometer's Sketchpad to construct a right triangle. Then, they change the measures of the legs to illustrate possible measures of side lengths. |
| Career Readiness Practices <br> CRP1. Act as a responsible and contributing citizen and employee. <br> CRP2. Apply appropriate academic and technical skills. <br> CRP4. Communicate clearly and effectively and with reason. <br> CRP6. Demonstrate creativity and innovation. <br> CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. <br> CRP9. Model integrity, ethical leadership and effective management. <br> CRP11. Use technology to enhance productivity. |
| 9.2 Career Awareness, Exploration, and Preparation Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Strand C: CareerPreparation9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, <br> and extracurricular activities for use in a career. |
| Career \& Technical Education Content Area: 21 ${ }^{\text {st }}$ Century Life and Careers Standards <br> 9.3.ST. 2 Use technology to acquire, manipulate, analyze and report data. <br> 9.3.ST-SM. 4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data. <br> 9.3.ST-SM. 3 Analyze the impact that science and mathematics has on society. |
| Interdisciplinary Connections: Trigonometry has many applications in Physics. |

Media Literacy:The students will research the proof by construction of the Pythagorean Theorem. They will include an explanation of the proof and accompanying drawings if necessary. Students will examine the historical and cultural significance of the
Pythagorean Theorem and its influence on the mathematics world at that time
III. Additional Differentiation/Modifications for Teaching

| Research-Based Effective Teaching Strategies | Additional Modifications for G\&T | Additional Strategies for Special Education | Additional Strategies for English Language Learners |
| :---: | :---: | :---: | :---: |
| Questioning techniques to facilitate learning <br> - See also Five Practices for Orchestrating Math Discussion <br> Math Discourse <br> - Talk Moves <br> Constructivist learning opportunities <br> - Piaget, Vygotsky, Bruner <br> Multiple Representations <br> Promote linguistic and nonlinguistic representations <br> Various types of feedback <br> - Student to student feedback <br> - Teacher to student feedback <br> Varied opportunities for students to communicate mathematically (orally, writing) <br> Use technological and /or physical tools (manipulatives) | See APEX for Classroom Differentiation for Gifted Students. | See APEX for Instruction for Classroom Differentiation for information on Special Need Students. | Extension: See APEX for Instruction for Scaffolding Instruction for English Language Learners. <br> ELD Standard <br> Standard 3 - Language of Mathematics English language learners communicate information, ideas and concepts necessary for academic success in the content area of mathematics. |

Algebra Lab
IV. Instructional Resources and Materials


Algebra Lab Pacing Guide

| Sept. | Oct. | Nov. | Dec. | Jan. | Feb. | March | April | May | June |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (20 days) <br> A.SSE.A. 1 <br> Combining <br> Like Terms, <br> Distributive <br> Property, <br> Factoring <br> Expressions <br> A.CED.A. 1 <br> A.REI.A. 1 <br> A.REI.B. 3 <br> A.CED.A. 4 <br> Explain how to and Solving <br> Linear <br> Equations <br>  <br> Graphing <br> Linear <br> Inequalities <br> A.CED.A. 4 <br> Literal <br> Equations <br> Linear <br> A.CED.A. 2 <br> A.REI.D. 10 | (20 days) <br> A.CED.A. 2 <br> Graphing Linear Inequalities A.REI.D. 11 Solutions to a system of equations <br> A.REI.D. 12 <br> Represent and solve systems of equations and inequalities (all methods) <br> A.CED.A. 3 A.SSE.A. 1 <br> Write/Interpret functions and the solutions to a system in terms of the context given F.IF.A. 1 F.IF.A. 3 <br> Construct and compare linear and exponential functions | (15 days) <br> Exponential <br> Properties <br> (including zero <br> and negative <br> exponents) <br> F.IF.B. 4 <br> F.IF.B. 5 <br> Graph functions (with/without technology) and interpret key features <br> F.IF.B. 6 <br> Analyze <br> functions represented in different ways | (15 days) <br> A.APR.A. 1 <br> A.SSE.A. 2 <br> Operations <br> with <br> Polynomials <br> A.REI.B. 4 <br> Solving <br> Quadratic <br> Functions <br> A.CED.A. 1 <br> F.IF.B.4* <br> F.IF.B.5* <br> Interpret and Graph <br> Quadratic <br> Functions <br> A.SSE.B. 3 <br> Factoring and <br> Completing the Square F.BF.A.1. <br> Describe quadratic relationships | (15 days) <br> G.GPE.B. 5 <br> Angles <br> Formed <br> within parallel lines $\frac{\text { G.CO.A. } 2}{\text { G.CO.B. } 6}$ <br> Perform translations reflections, rotations <br> Understand congruency as related to isometries <br> Define dilations | (15 days) <br> G.CO.A.3-5. <br> Perform and define transformations that map onto themselves <br> Rotational and reflectional symmetry of 2-D figures <br> G.CO.B.7-8 <br> Identify triangle congruence by SSS, SAS, ASA, AAS, and HL <br> G.CO.D.12-13 <br> Perform <br> Constructions | (20 days) <br> G.SRT.A. 1 <br> Perform <br> dilations <br> w/various centers <br> G.SRT.A.2-3 <br> Identify <br> triangle <br> similarity <br> using AA, <br> SAS, and SSS <br> G.CO.C.9-10. <br> 2-column <br> algebraic <br> proofs and <br> proofs of lines and angles | (20 days) G.CO.C. $9-10$. Mid-segment Median G.CO.C. 11 G.GPE.B. 4 Properties of parallelograms Proving parallelogram theorems G.SRT.B. 4 Side Splitter Theorem | (15 days)G.SRT.B.5.2 -column <br> triangle <br> congruence <br> proofs usingSSS, SAS, <br> ASA, AAS, HL <br> and CPCTCG.SRT.B.5.2-columnsimilarity proofsusing AA, SAS,and SSSG.GPE.B. 6G.GPE.B. 7PythagoreanTheoremDistance/Midpoint | (15 DAYS) <br> G.SRT.C.6. <br> Trig Ratios <br> G.SRT.C. 7 <br> G.SRT.C. 8 <br> Co-function <br> Identities <br> Solving <br> Right <br> Triangles <br> Angle of <br> Elevation and <br> Depression |

## Algebra Lab

